Stat 212b: Topics in Deep Learning
Lecture 4

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DeepMind makes Nature Cover

DeepMind designed an algorithm that beat a professional GO player for the first time, using MCTS and two CNNs trained with supervised learning and reinforcement learning.

[http://www.nature.com/news/google-ai-algorithm-masters-ancient-game-of-go-1.19234]

- Thus $\Phi(x) = |Vx|$ satisfies

$$\boxed{\forall\ x, t\ ,\ \ \Phi(\varphi_t(x)) = \Phi(x)\ .}$$

- Indeed,

$$A = V^* \mathrm{diag}(\lambda_1, \ldots, \lambda_n) V \implies e^{itA} = V^* \mathrm{diag}(e^{it\lambda_1}, \ldots, e^{it\lambda_n}) V\ .$$

$$\begin{aligned} V\varphi_t x &= V e^{itA} x = V V^* \mathrm{diag}(e^{it\lambda_1}, \ldots, e^{it\lambda_n}) V x \\ &= \mathrm{diag}(e^{it\lambda_1}, \ldots, e^{it\lambda_n}) V x \end{aligned}$$

$$\text{thus } \Phi(\varphi_t x) = |V\varphi_t x| = |Vx|\ .$$

Review: Limits of Group Diagonalisation

- A shallow (1 layer) network is thus sufficient to achieve invariance to commutative group transformations:

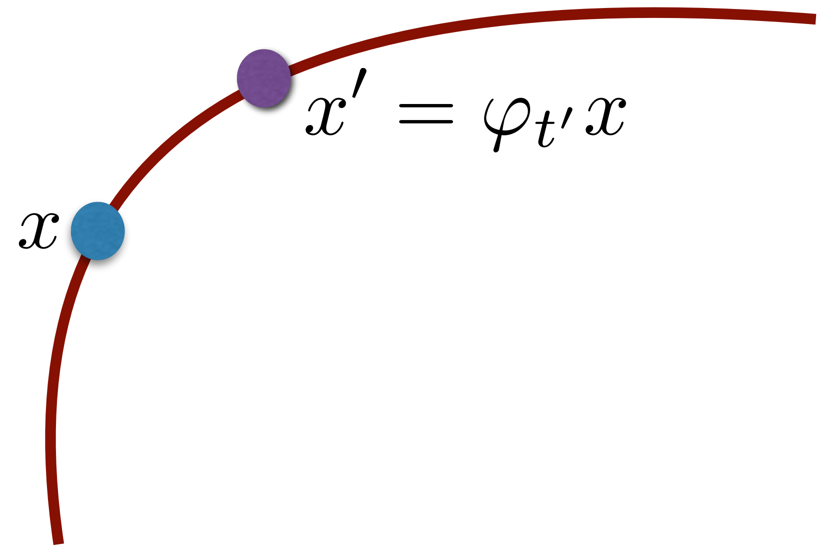
$$x \longrightarrow \boxed{V} \longrightarrow \boxed{|z|} \longrightarrow \Phi(x)$$

- However, this architecture has a number of shortcomings.
    - Not applicable to non-commutative, discrete symmetry groups
    - Not discriminative in general
    - Not stable

Objectives

- Wavelets
- Point-Wise non-linearities
- Scattering Representations for the Translation Group
- Properties

Local invariants and convolution

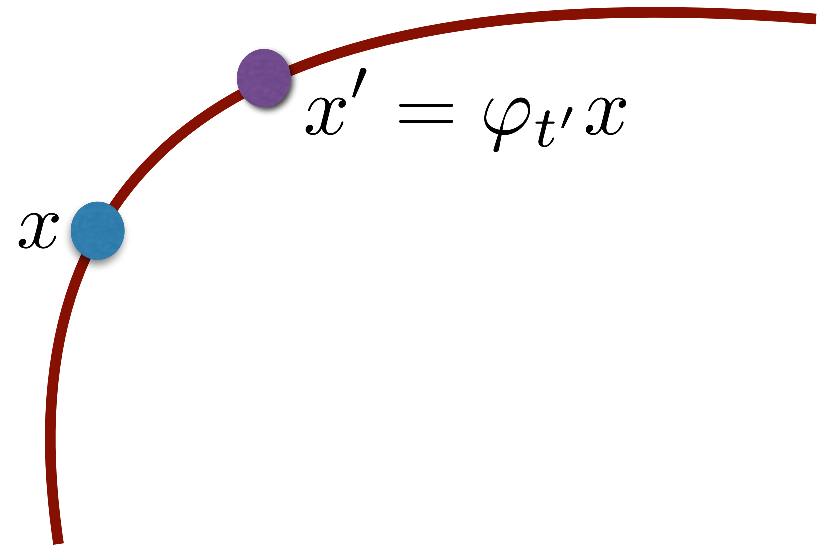


- Local translation invariance:

$$\|\Phi(x) - \Phi(\varphi_v x)\| \leq C 2^{-J} \|v\| \text{ , or}$$

$$\forall\ v, \|x\| = 1\ ,\ \frac{\|\Phi(x) - \Phi(\varphi_v x)\|}{\|v\|} \leq C 2^{-J}\ .$$

Local invariants and convolution



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$$\forall\ v, \|x\| = 1\ ,\ \frac{\|\Phi(x) - \Phi(\varphi_v x)\|}{\|v\|} \leq C 2^{-J}\ .$$

- So, we want to *smooth* along the orbits.
- Local averaging within the translation orbit:

$$\Phi(x) = 2^{-dJ} \int_v \phi(2^{-J} v) \varphi_v x dv\ ,\ \left(\int \phi(v) dv = 1, \phi \geq 0 \right)\ .$$

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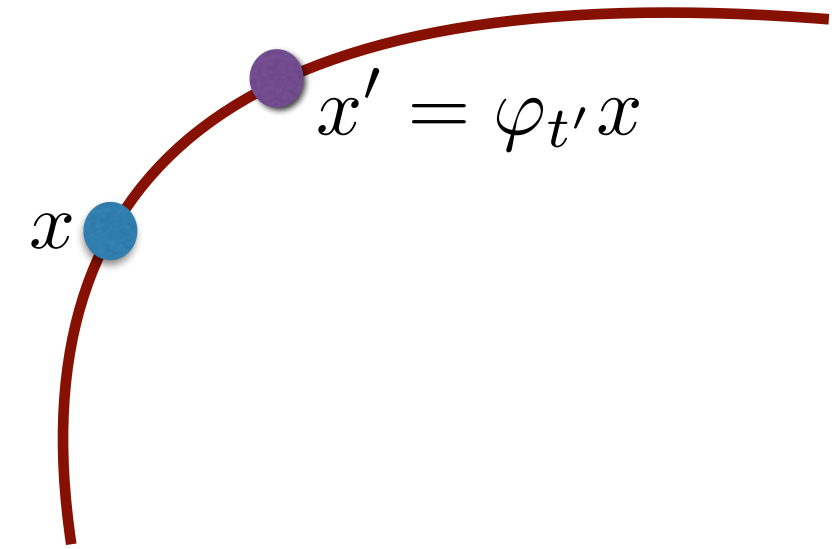

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- In coordinates, it becomes

$$\Phi(x)(u) = \int \phi_J(v) x(u-v) dv = x * \phi_J(u) \ , \text{with}$$

$$\phi_J(v) = 2^{-Jd} \phi(2^{-J} v)$$

Local average and stability

Proposition: The local averaging $\Phi(x) = x * \phi_J$ satisfies
$\forall \ \|x\| = 1 \in L^2 \ , \tau \ , \ \|\Phi(x) - \Phi(\varphi_\tau x)\| \leq C \|\tau\| \ .$

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- Not surprising, since this operator removes the problematic high-frequencies.
- Are there other linear operators with the same property?

Average and uniqueness

- The only linear, translation-invariant operator is the average:

$$\forall\ v\ ,\ \ \Phi(x) = \Phi(\varphi_v x) \Longrightarrow \Phi(x) = \frac{1}{|G|}\int \Phi(\varphi_v x) dv$$

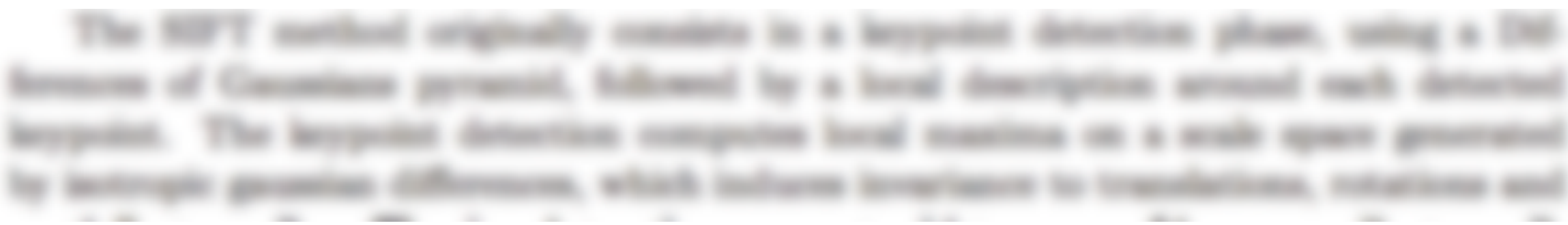
$$\Longrightarrow \Phi(x) = \Phi\left(\frac{1}{|G|}\int \varphi_v x dv\right) = \Phi\left(\frac{1}{|G|}\int x(u) du\right)\ .$$

- And a similar argument can be used locally.

From averages to Wavelets

- Low-pass information is insufficient:

The SIFT method originally consists in a keypoint detection phase, using a Differences of Gaussians pyramid, followed by a local description around each detected keypoint. The keypoint detection computes local maxima on a scale space generated by isotropic gaussian differences, which induces invariance to translations, rotations and



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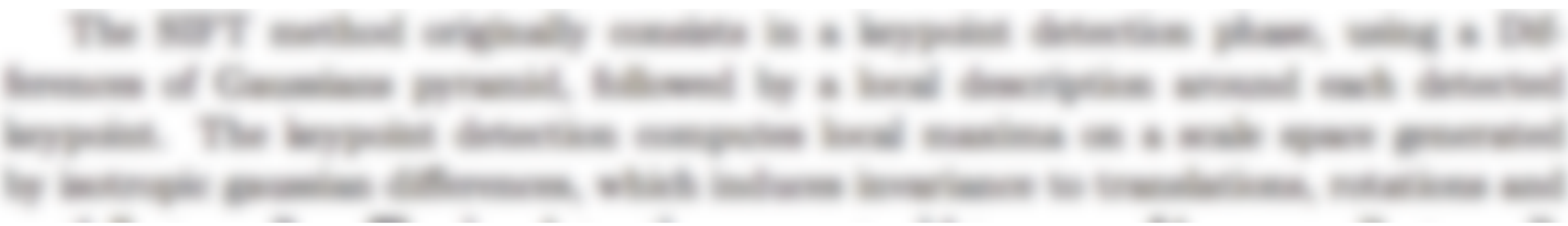

- Thus, we must capture high-frequency.
- These new measurements must involve a non-linearity.

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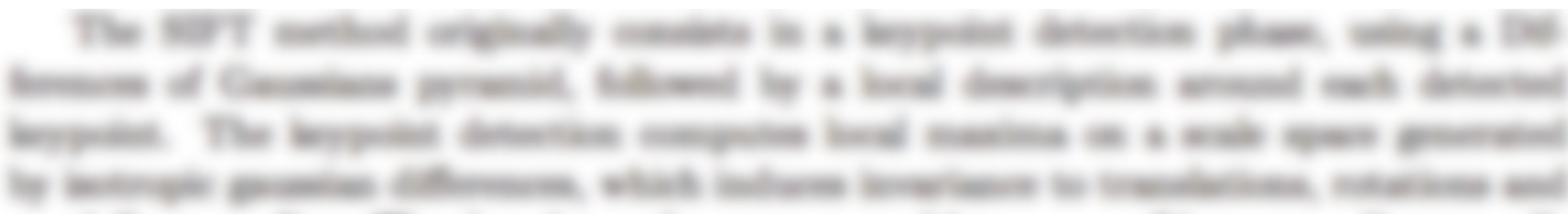
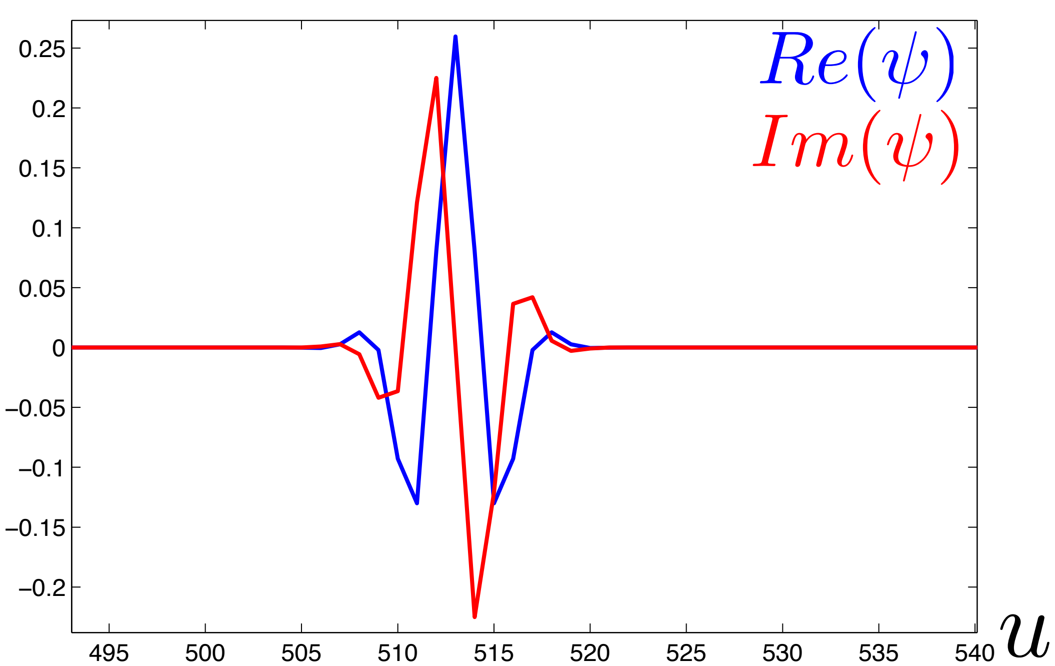


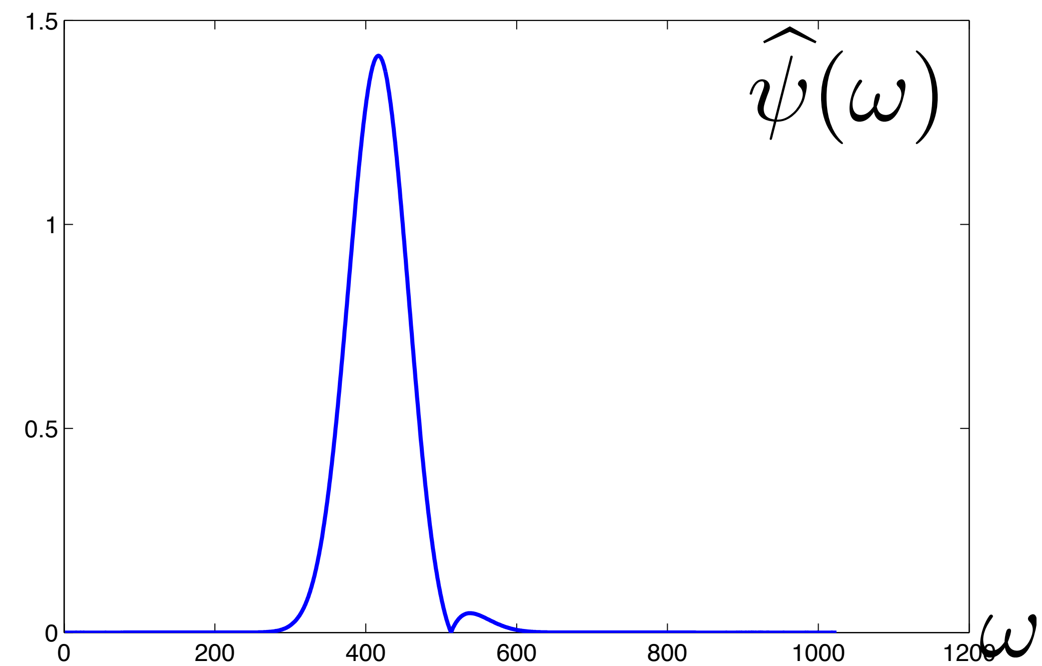
- Thus, we must capture high-frequency.
- These new measurements must involve a non-linearity.
- We want them to preserve stability to deformations.
- And we want them to preserve inter-class variability.

Wavelets

- ψ : bandpass (ie oscillating) signal, well localized in space and frequency.

Ex: Morlet wavelet





Wavelets

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- At least one vanishing moment: $\int \psi(u) du = 0$

(we say that ψ has k vanishing moments if $\int \psi(u) u^l du = 0$ for $l < k$)

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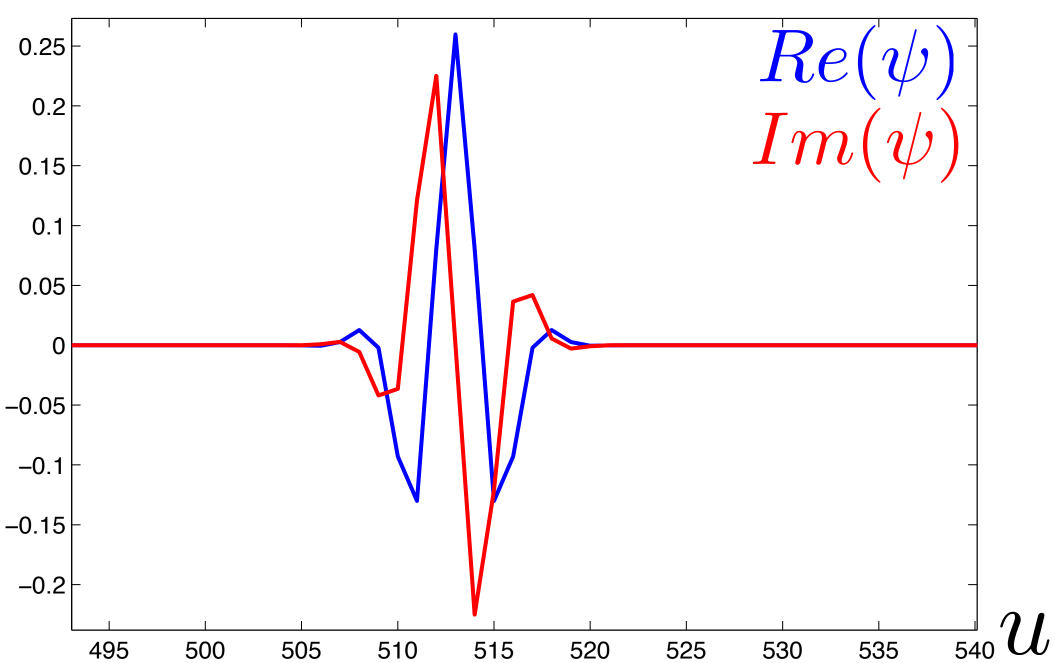

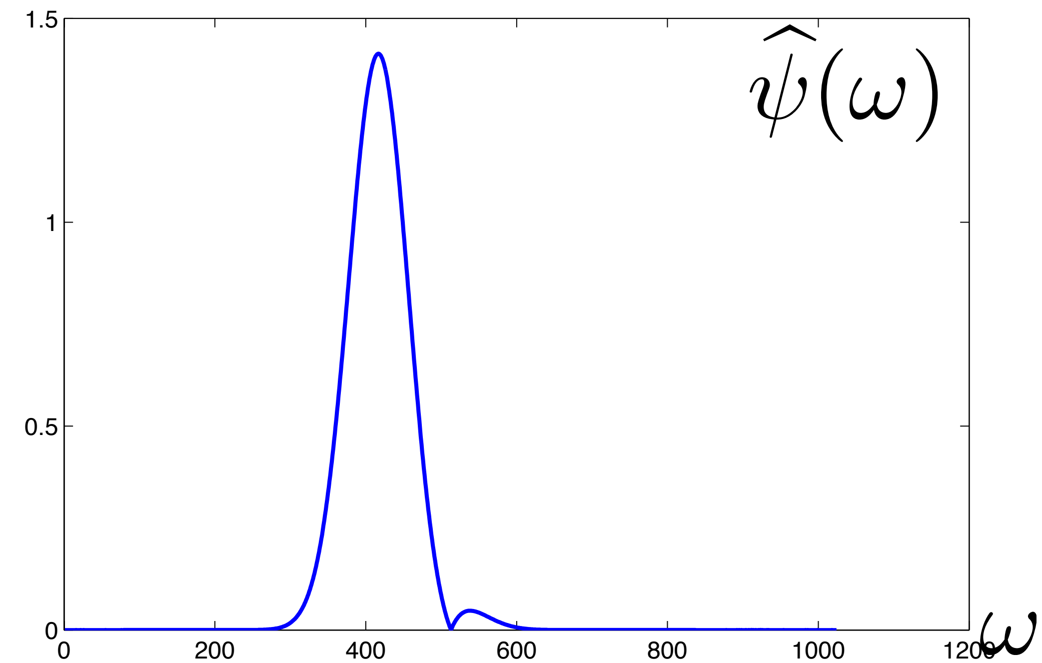

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- Can be real or complex. $\psi = \psi_r + i\psi_i$

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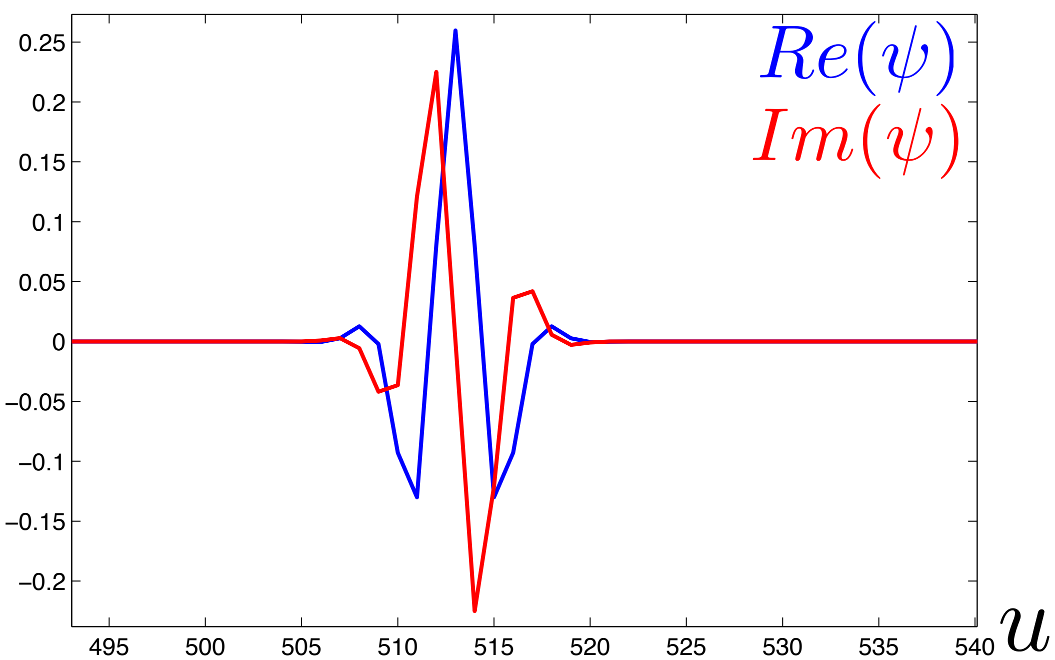

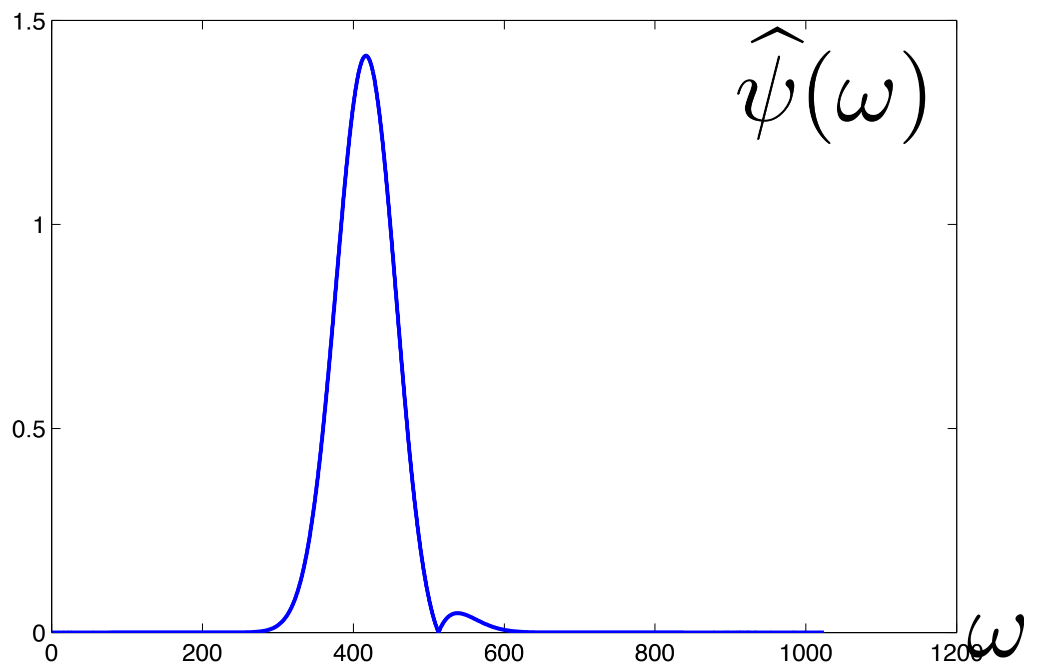

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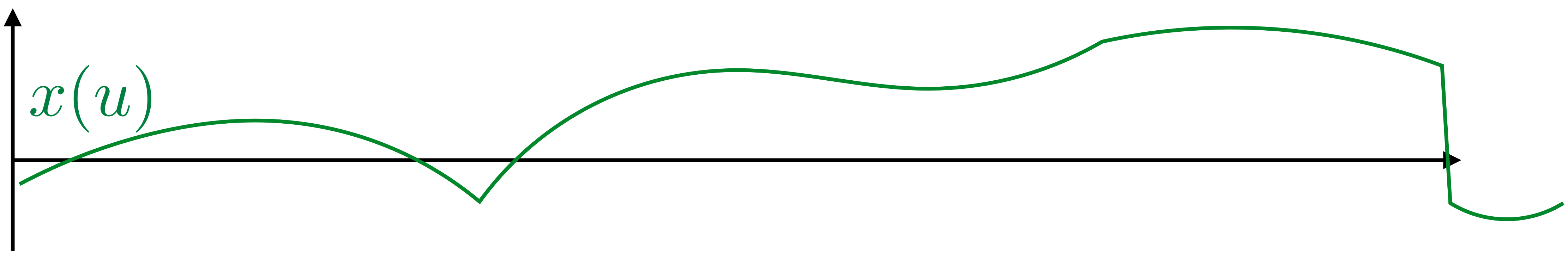
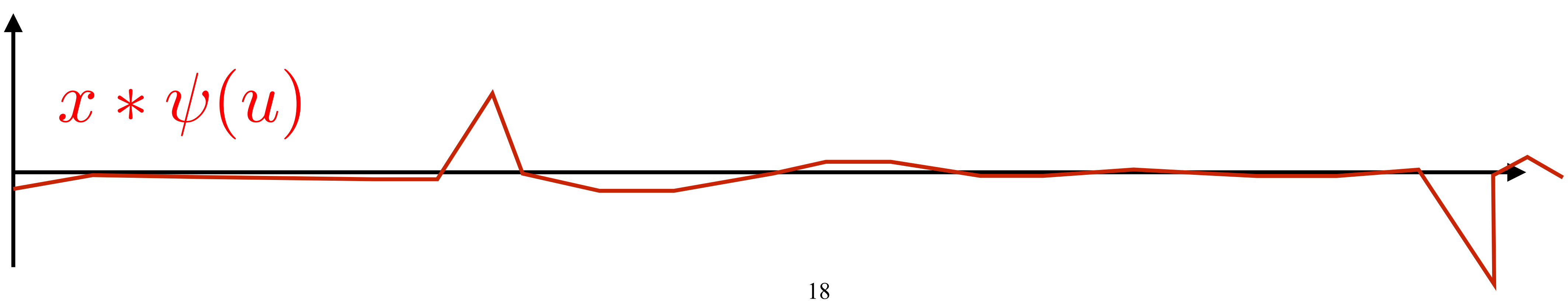
- If $x(u)$ is piece-wise smooth, then $x * \psi(u)$ is mostly zero





Wavelets

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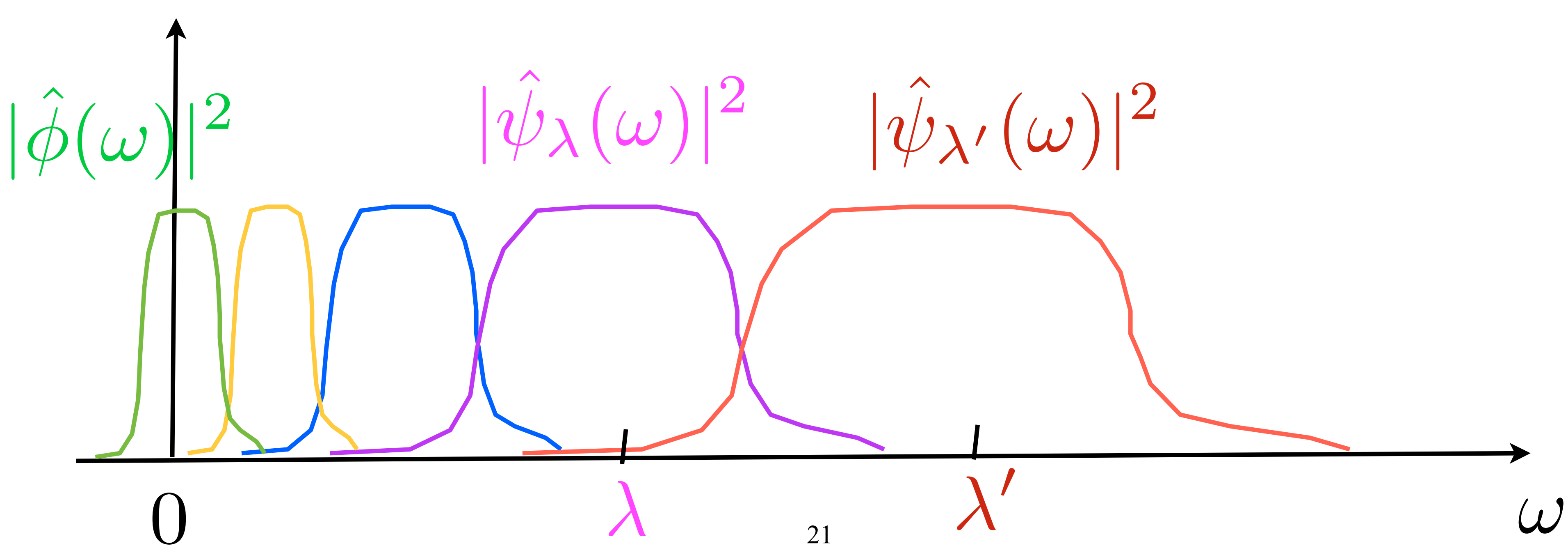
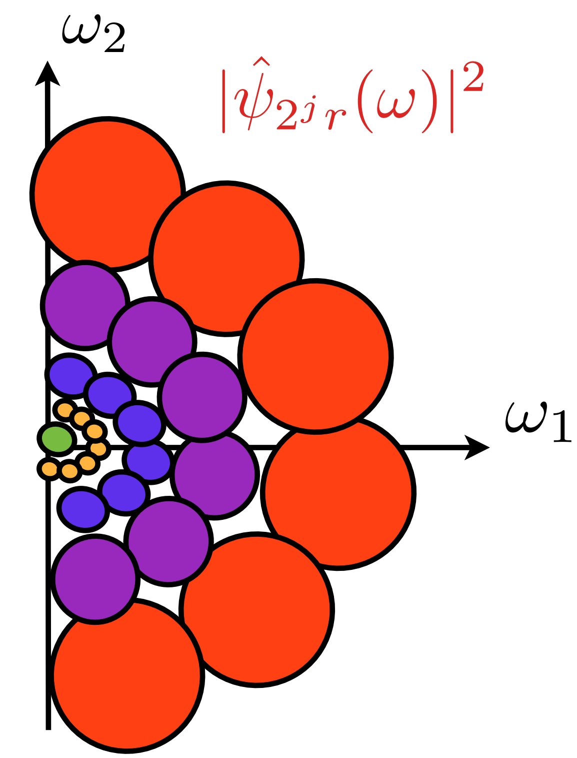
- Dilated wavelets: $\psi_j(u) = 2^{-j}\psi(2^{-j}u)\,,\ j \in \mathbb{Z}$



Littlewood-Paley Wavelet Filter Banks

- For images, dilated and rotated wavelets:

$$\psi_\lambda(u) = 2^{-j/2}\psi(2^{-j}ru) \ , \text{ with } \lambda = 2^j r$$



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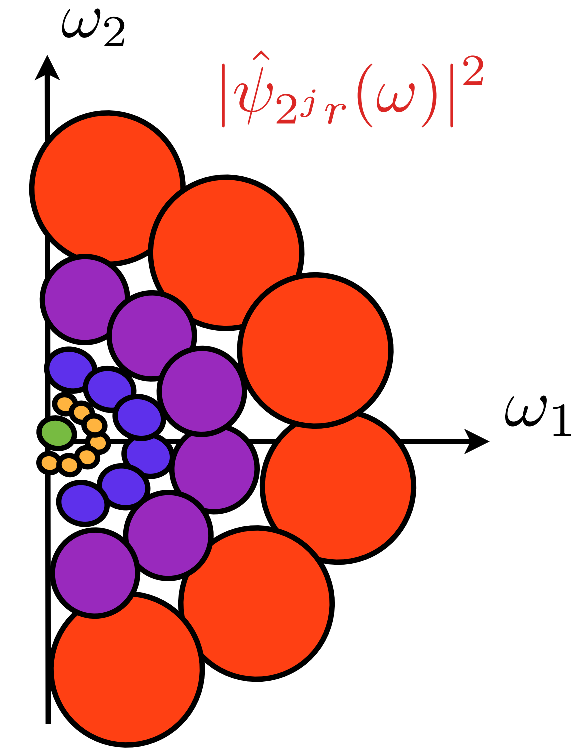

- Wavelet transform convolutional filter bank:

$$Wx = \{x \star \phi(u)\ ,\ x \star \psi_\lambda(u)\}_{\lambda \in \Lambda}$$

$$x \star \psi(u) = \int x(v)\psi(u-v)dv\ .$$

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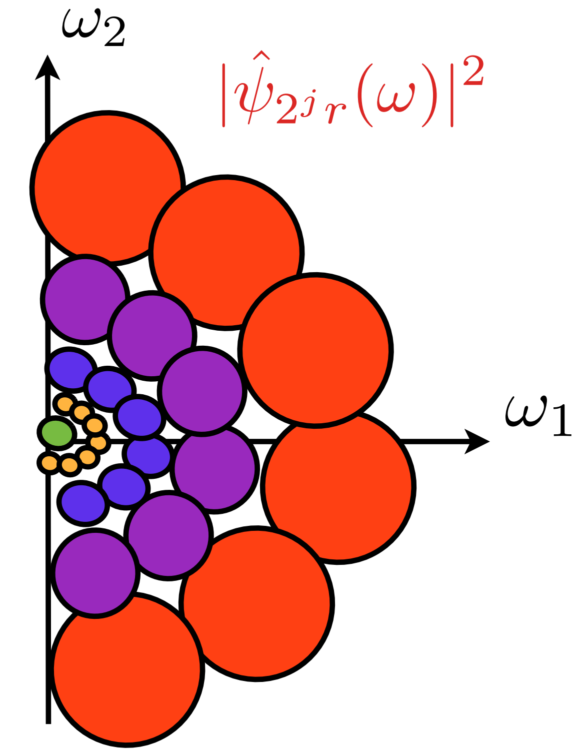

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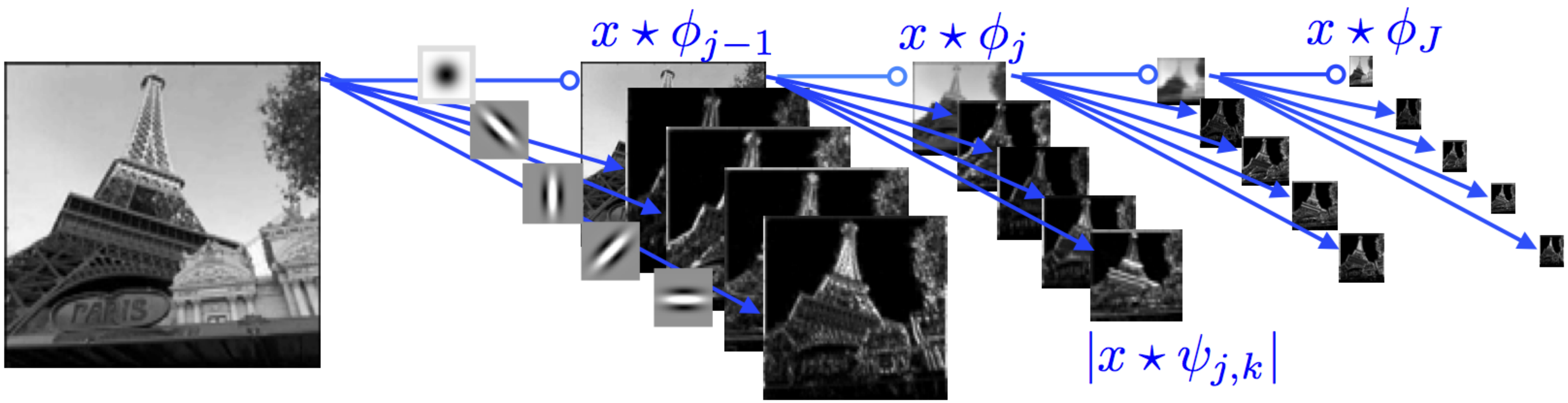
Theorem (Littlewood-Paley): If there exists $\delta > 0$ such that

$$\forall \omega > 0\ ,\ 1-\delta \leq |\hat{\phi}(\omega)|^2 + \frac{1}{2}\sum_\lambda |\hat{\psi}(\lambda^{-1}\omega)|^2 \leq 1\ ,$$

then $\forall\, x \in L^2$, $(1-\delta)\|x\|^2 \leq \|Wx\|^2 \leq \|x\|^2$.

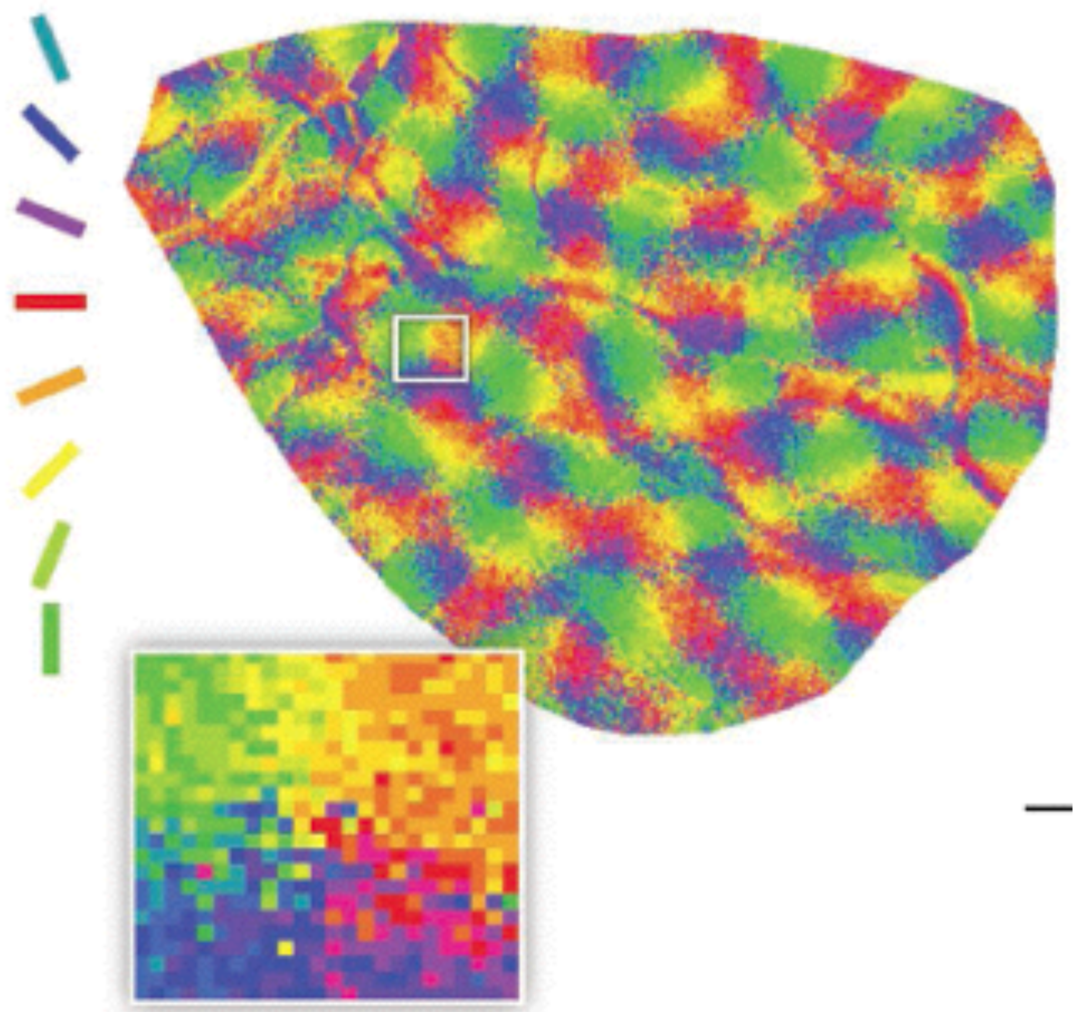
Wavelet Filter Banks

- We can compute a wavelets recursively in a fine-to-coarse transform:



Wavelets in Vision

- *V1* Model of Simple and Complex cells: First layer of processing is selective in orientation, scale and position.



  - cells are organized in *pinwheels*. (more on that later).

Why are wavelets a good model?

- We will see that they provide stability to deformations because they *commute* nicely with diffeomorphisms:

$$\|W\varphi_\tau x - \varphi_\tau W x\| \lesssim \|\tau\| \ .$$

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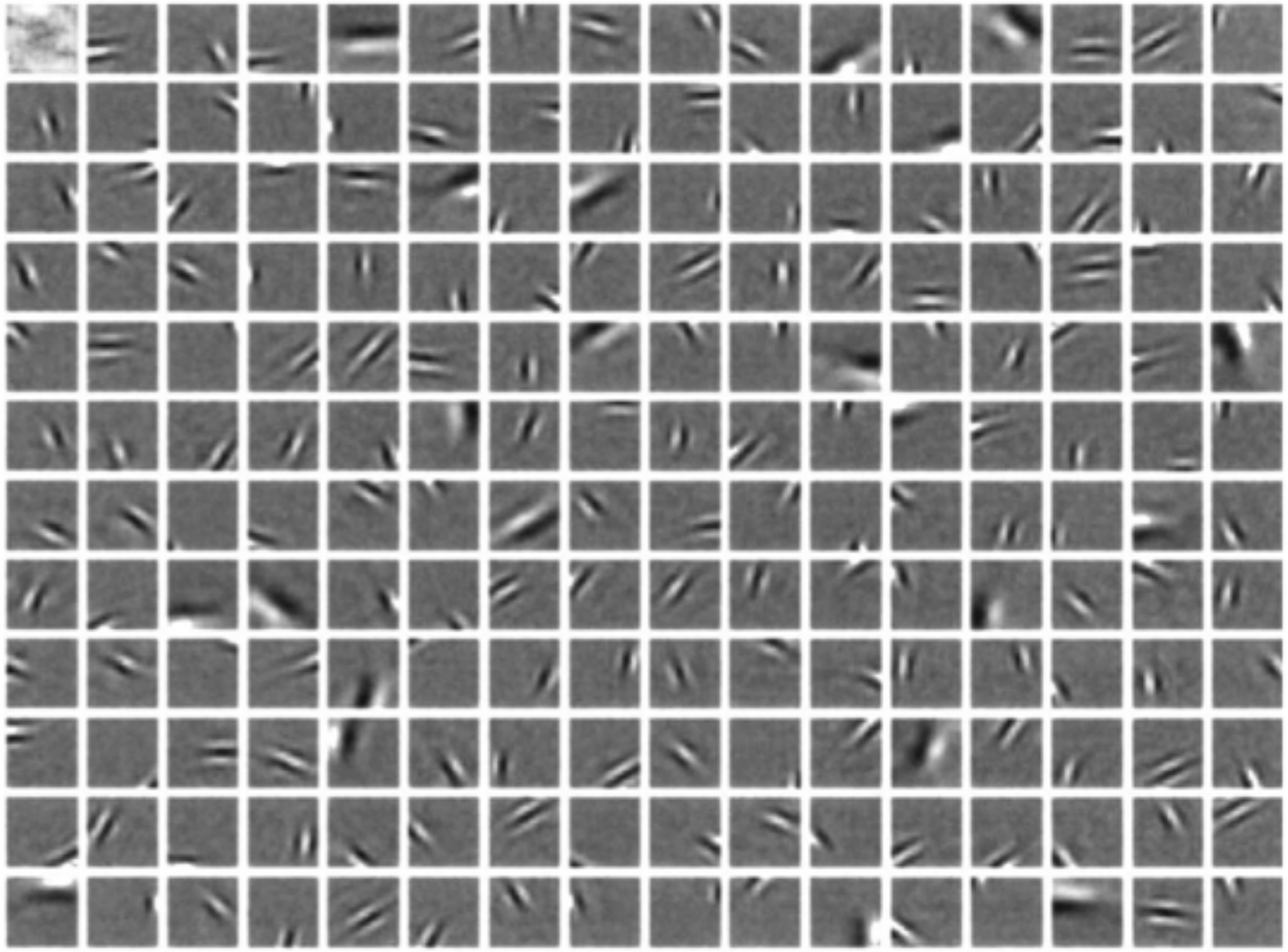
- We will also see that the discriminability of $\Phi(x) = \rho(Wx)$ is controlled by the *sparsity* produced by W :

$$\{x * \psi_\lambda(u)\}_{\lambda,u} \text{ has few non-zero coefficients.}$$

Examples

- Olshausen and Field Sparse coding model trained on natural images:



$$\min_{W,z} \|X - Wz\|^2 + \lambda \|z\|_1$$

[Olshausen and Field,'96]

Examples

- Top performing shallow network unsupervised learning:

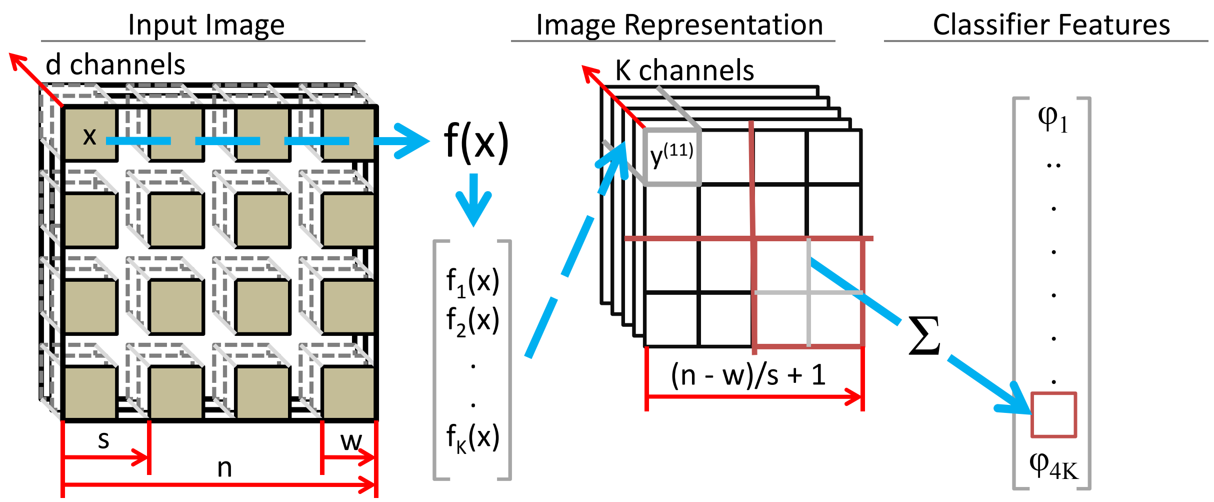
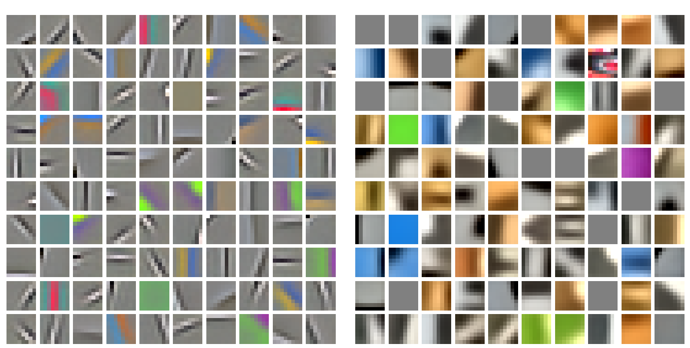
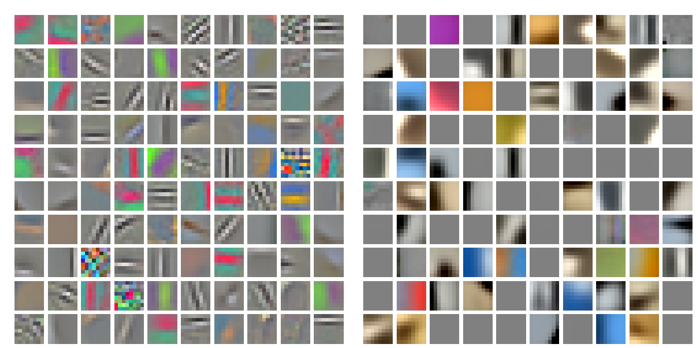


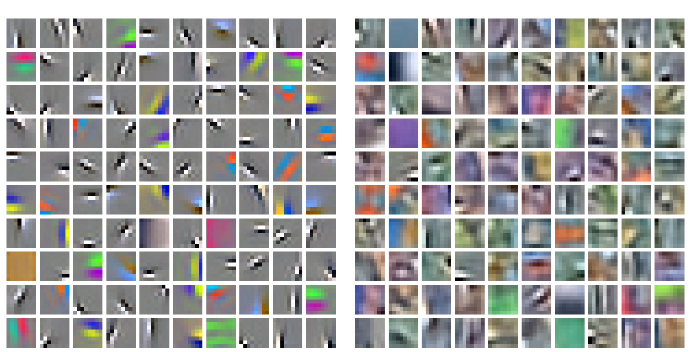
Figure 1: Illustration showing feature extraction using a w-by-w receptive field and stride s. We first extract w-by-w patches separated by s pixels each, then map them to K-dimensional feature vectors to form a new image representation. These vectors are then pooled over 4 quadrants of the image to form a feature vector for classification. (For clarity we have drawn the leftmost figure with a stride greater than w, but in practice the stride is almost always smaller than w.



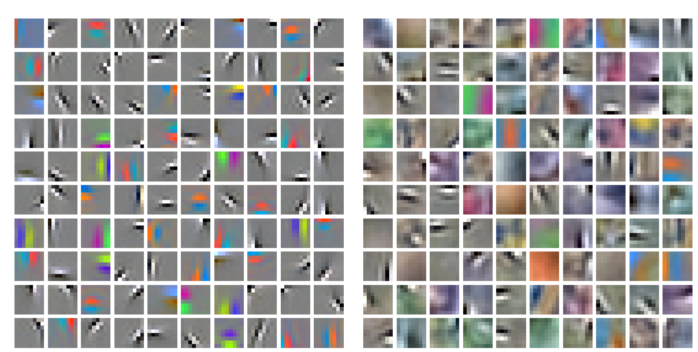
(a) K-means (with and without whitening)



(b) GMM (with and without whitening)



(c) Sparse Autoencoder (with and without whitening)



(d) Sparse RBM (with and without whitening)

[Coates, Le & Ng, '10]

Wavelets and Deformations

- We saw before that a blurring kernel is nearly invariant to deformations:

Proposition: The local averaging $\Phi(x) = x * \phi_J$ satisfies
$\forall \ \|x\| = 1 \in L^2 \ , \ \tau \ , \ \|\Phi(x) - \Phi(\varphi_\tau x)\| \leq C \|\tau\| \ .$

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- What about the wavelet operator $\Phi(x) = \{x * \psi_\lambda\}_\lambda$?
  - We don't have local invariance, but we have a form of local covariance:

Proposition [Mallat]: For each $\delta > 0$ there exists $C > 0$ such that for all J and all $\tau \in C^2$ with $\|\nabla \tau\|_\infty \leq 1 - \delta$ we have

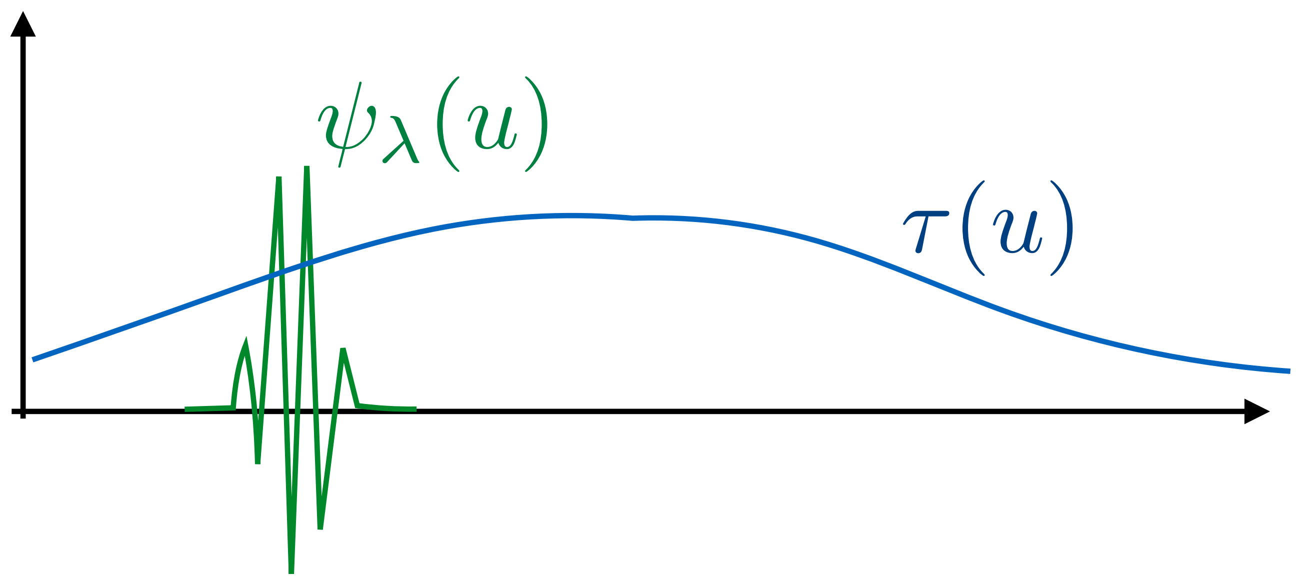
$$\|W_J \varphi_\tau - \varphi_\tau W_J\| \leq C (J \|\nabla \tau\|_\infty + \|H\tau\|_\infty) \ .$$

($H\tau$: Hessian of τ)

Wavelets and Deformations

- Qualitative idea behind this result:

Each ψ_λ only "sees" the part of the deformation τ that intersects its support.



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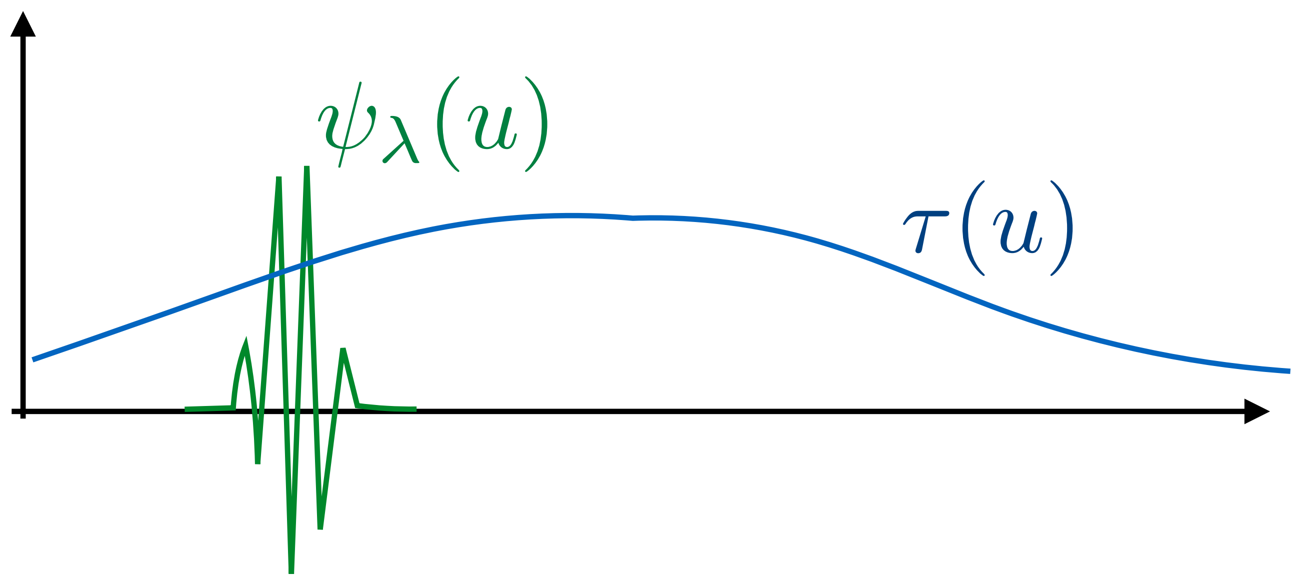

For small scales, ψ_λ has small support, and for u, v within that support, because τ is smooth, $|\tau(v) - \tau(u)| \sim 2^{-j} |\nabla \tau|_\infty$.

Thus $|(\varphi_\tau x) * \psi_\lambda(u) - x * \psi_\lambda(u - \tau(u))| \sim |\nabla \tau|_\infty$.

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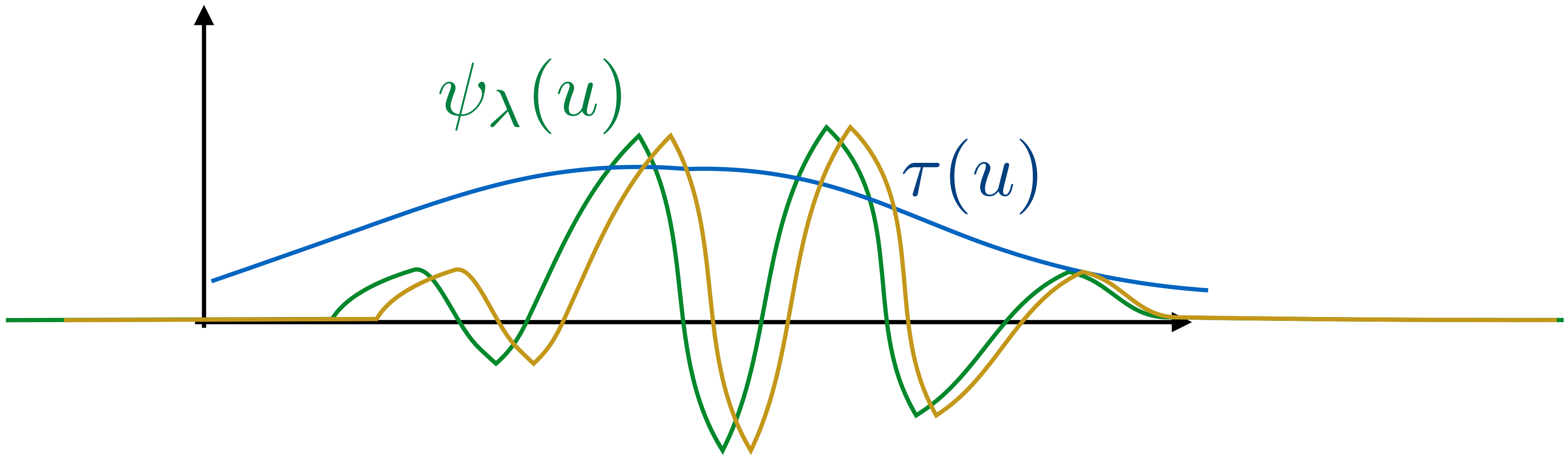

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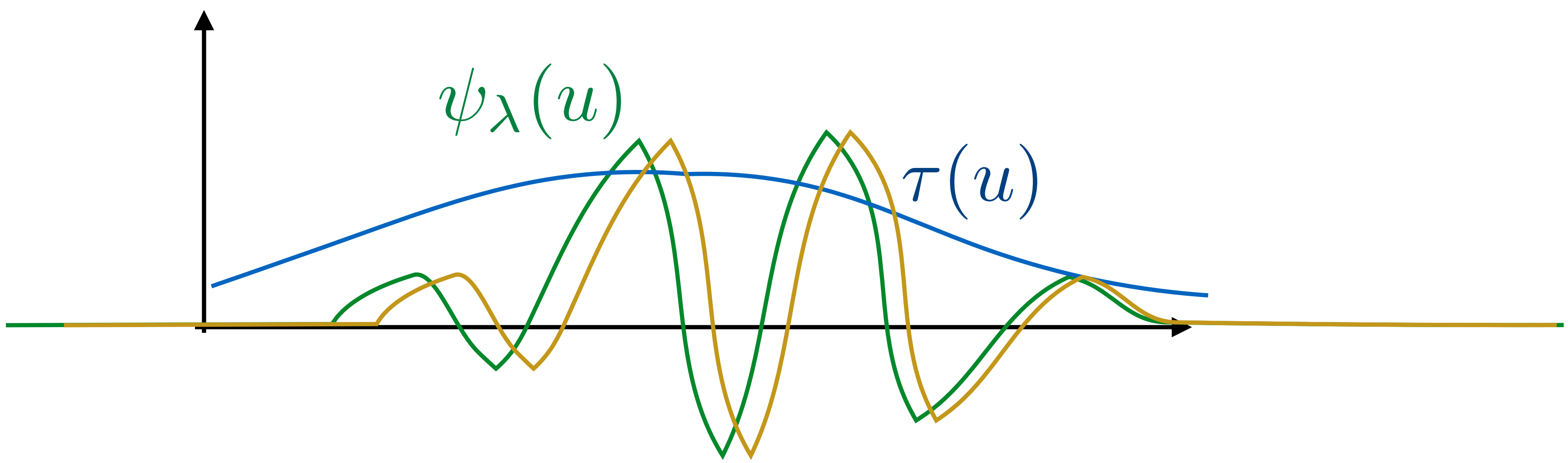

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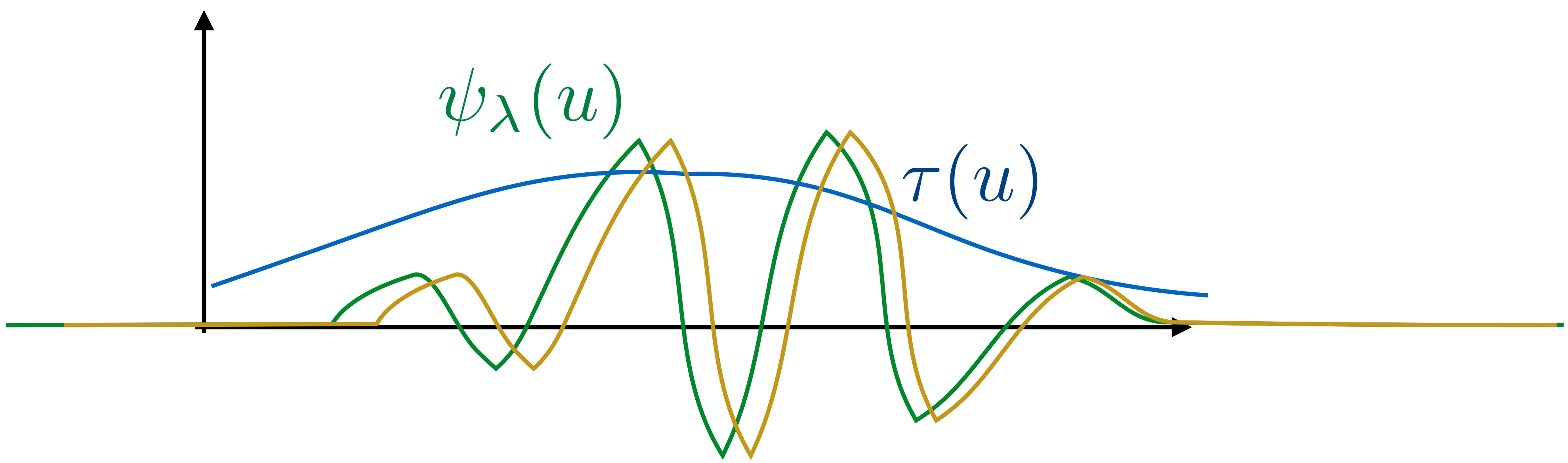

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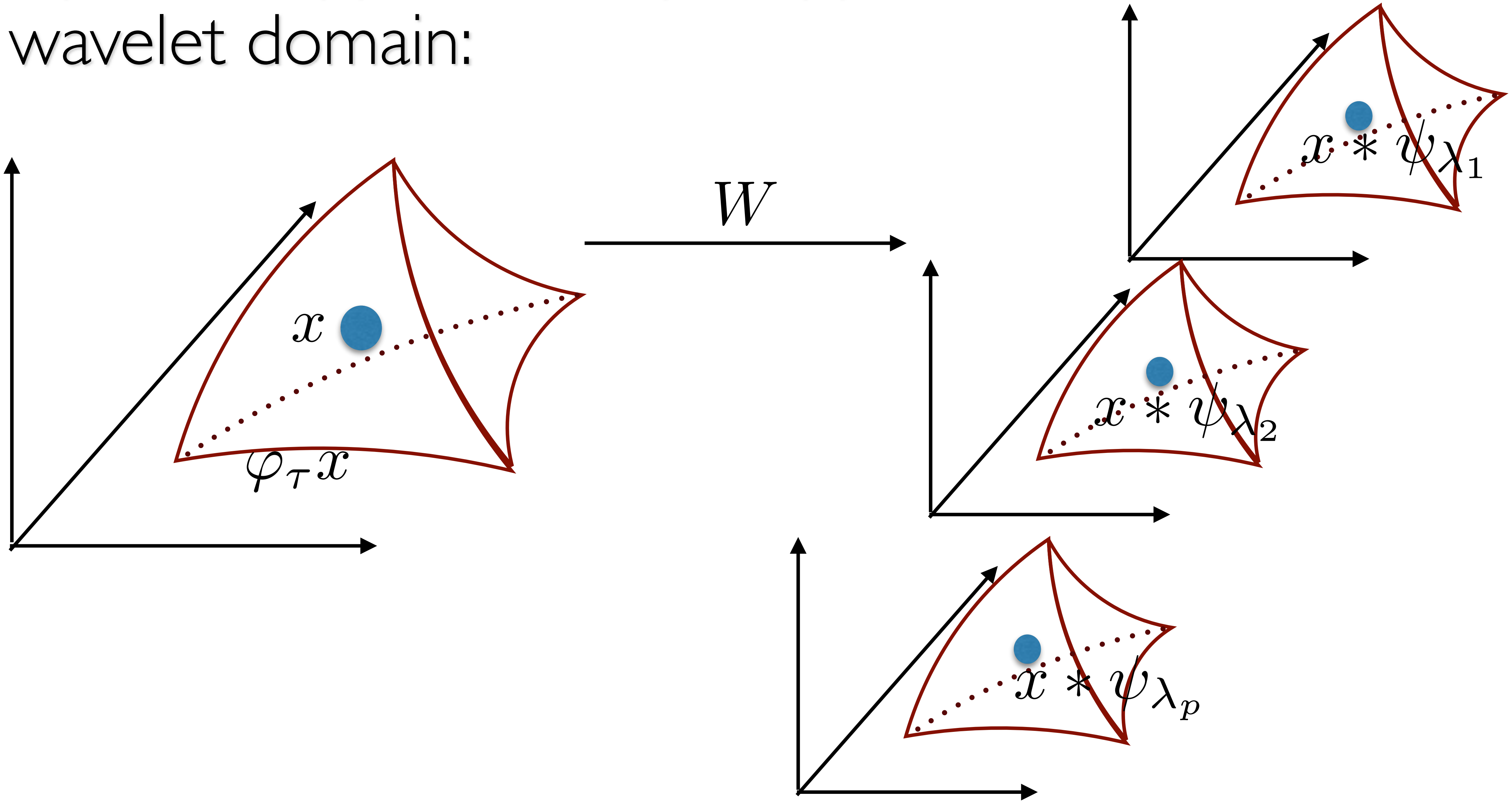
And, most importantly, wavelet separates scales
(so errors do not accumulate)

Wavelets and Non-linearities

- The commutation property says that deformations in the input are approximately mapped to deformations in the wavelet domain:



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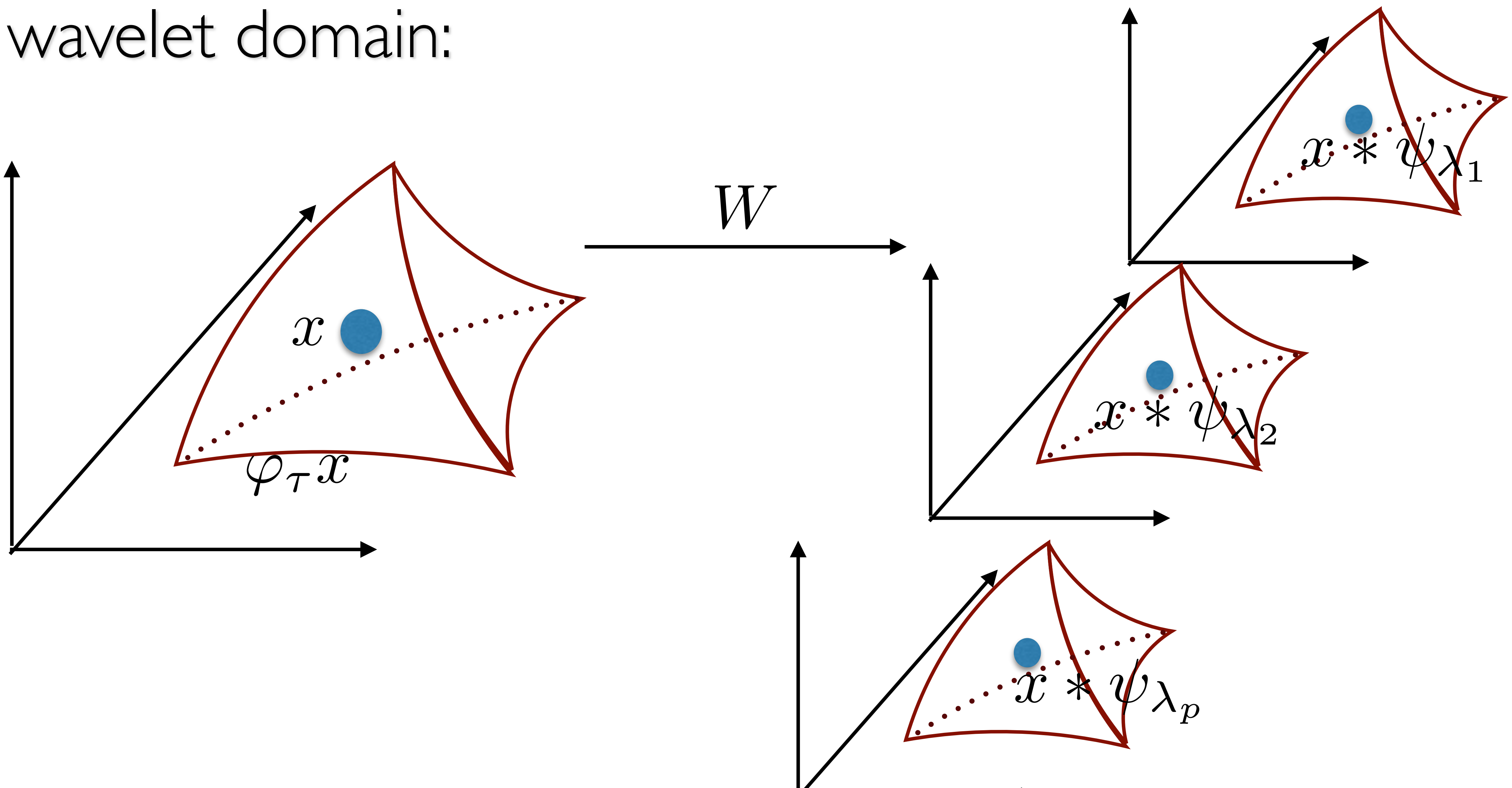

- We want to extract again stable measurements: *need non-linear operator.*

Characterization of stable non-linearities

- Preserve additive stability:

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- Preserve geometric stability: It is sufficient to commute with diffeomorphisms:

$$\left.\begin{array}{l} \Phi \text{ stable: } \|\Phi(\varphi_\tau x) - \Phi(x)\| \lesssim \|\tau\| \\ M \text{ commutes with } \varphi_\tau \ \forall \tau. \end{array}\right\} \Rightarrow$$

$M\Phi$ and ΦM stable:

$$\|\Phi M(\varphi_\tau x) - \Phi M(x)\| \lesssim \|\tau\|$$

$$\|M\Phi(\varphi_\tau x) - M\Phi(x)\| \lesssim \|\tau\|$$

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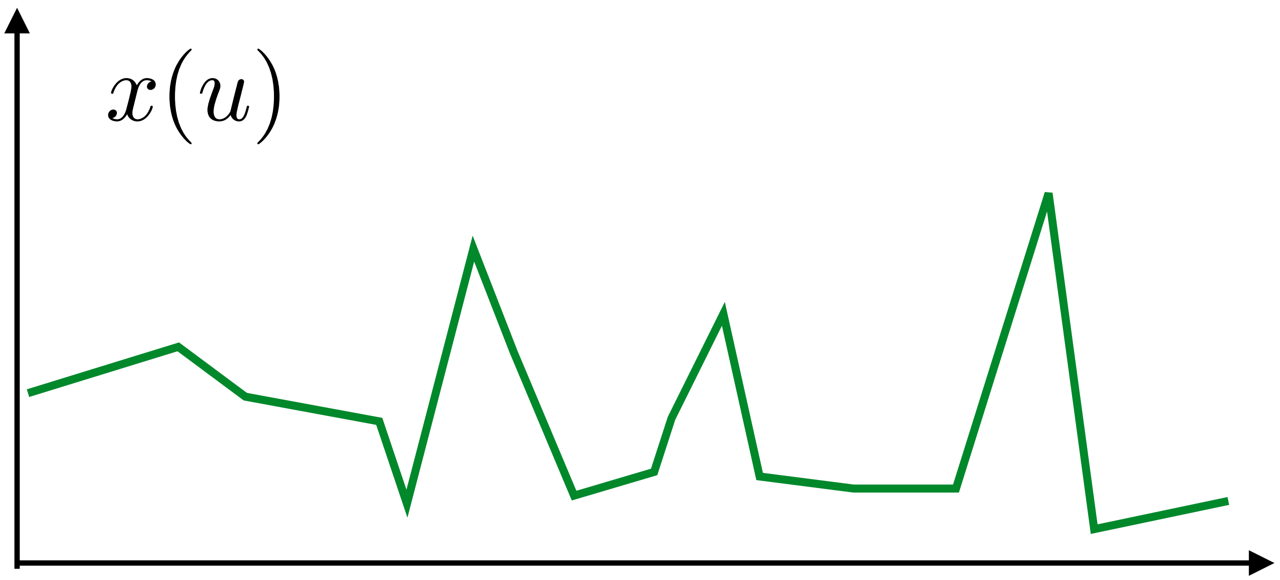
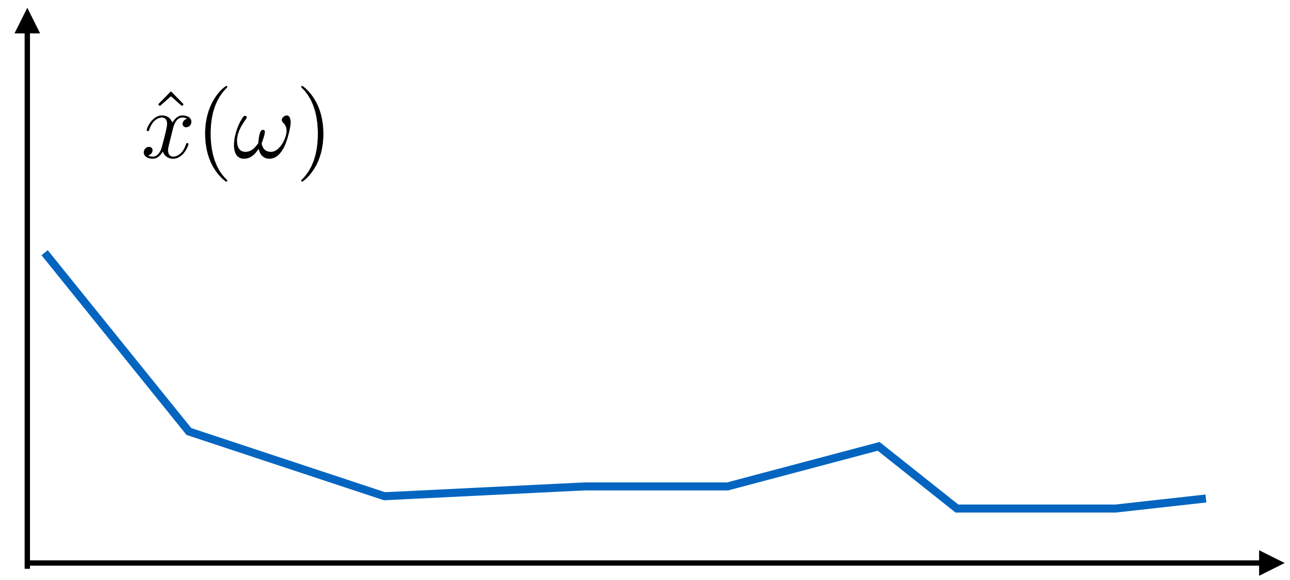
- Since we want to smooth orbits, we may choose a point-wise nonlinearity that reduces oscillations:

$$\rho(z) = |z| \text{ or } \rho(z) = \max(0, z)$$

Understanding the effect of nonlinearities

- Rectifiers thus perform a *non-linear demodulation*:





Understanding the effect of nonlinearities

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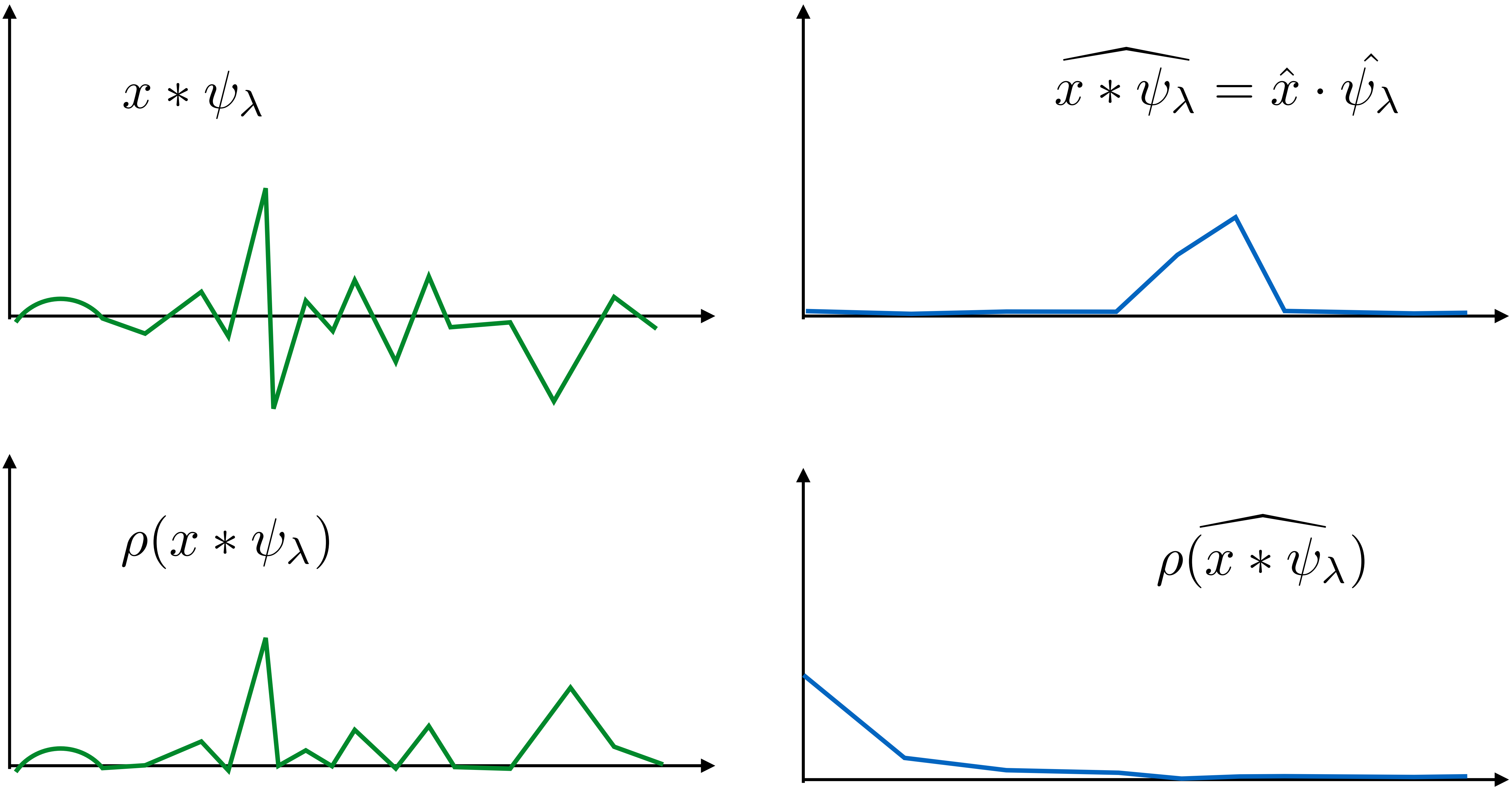

sometimes called the *envelope*

Choice of Pointwise Nonlinearity

- Full rectification $\rho(z) = |z|$ preserves energy:
  - When the wavelet is complex, it produces smoother envelopes (thus more stable features).

- Half rectification (ReLU) $\rho(z) = \max(z, 0)$ captures half the energy, and it also creates *sparsity.*
  - We will see that this is important to perform *detection.*

- Sigmoid nonlinearity $\rho(z) = (1 + e^{-z})^{-1}.$
  - It is not homogeneous
  - Saturating regimes are problematic for learning via back propagation in deep models.

- "Leaky" ReLU [MSR'14]: parametrized half-rectifier.

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- Local averaging kernel: $x \star \phi_J$
  - locally translation invariant
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  - loss of high-frequency information.

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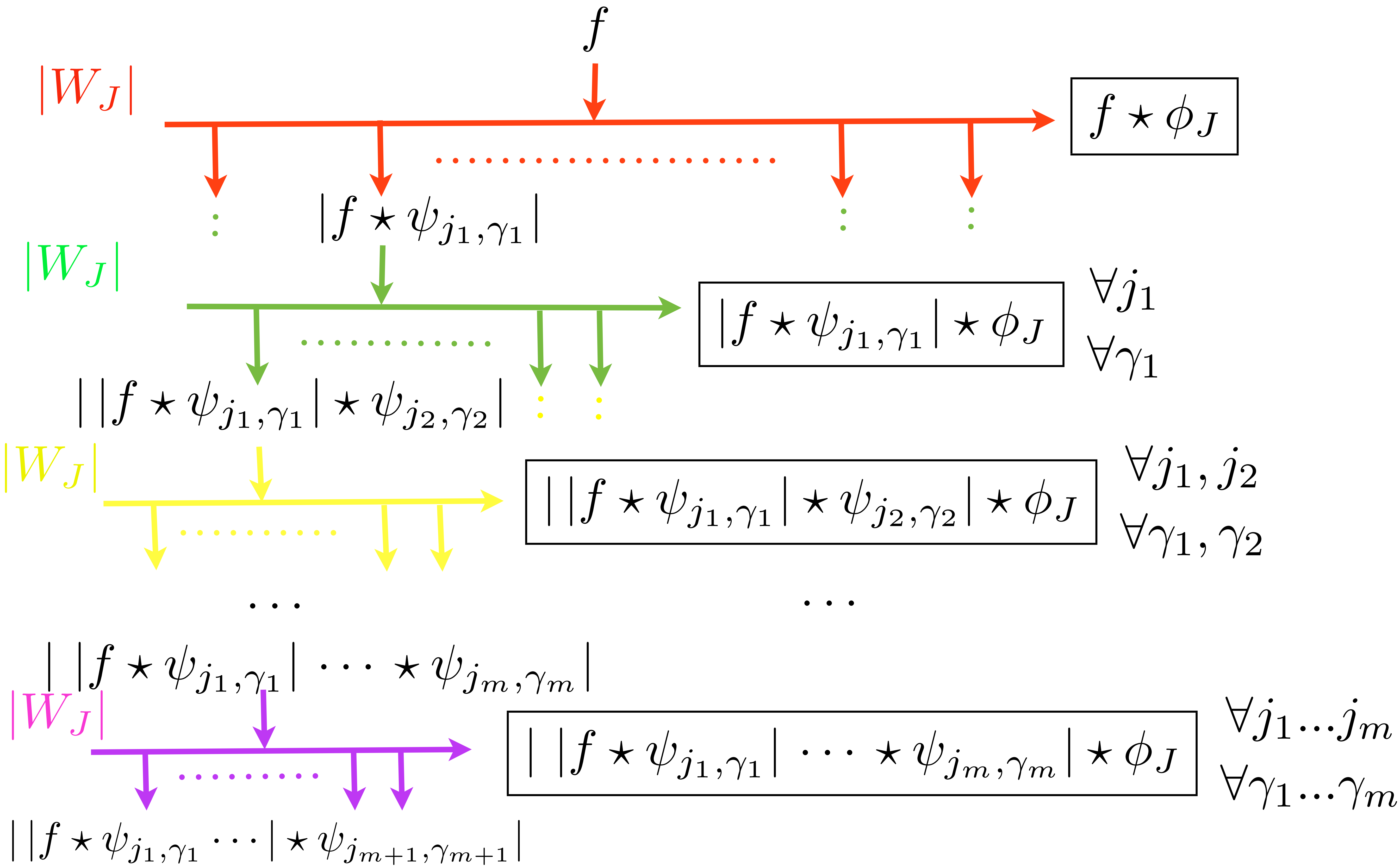
- Scattering coefficient along a path $p = (\lambda_1, \ldots, \lambda_m)$:

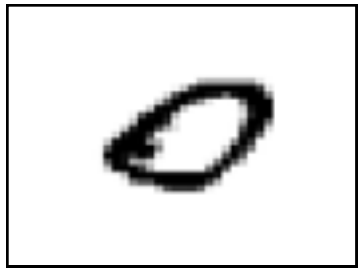
$$S_J[p]x(u) = |||x \star \psi_{\lambda_1}| \star \psi_{\lambda_2}| \star \ldots | \star \psi_{\lambda_m}| \star \phi_J(u) \ .$$

Scattering Convolutional Network



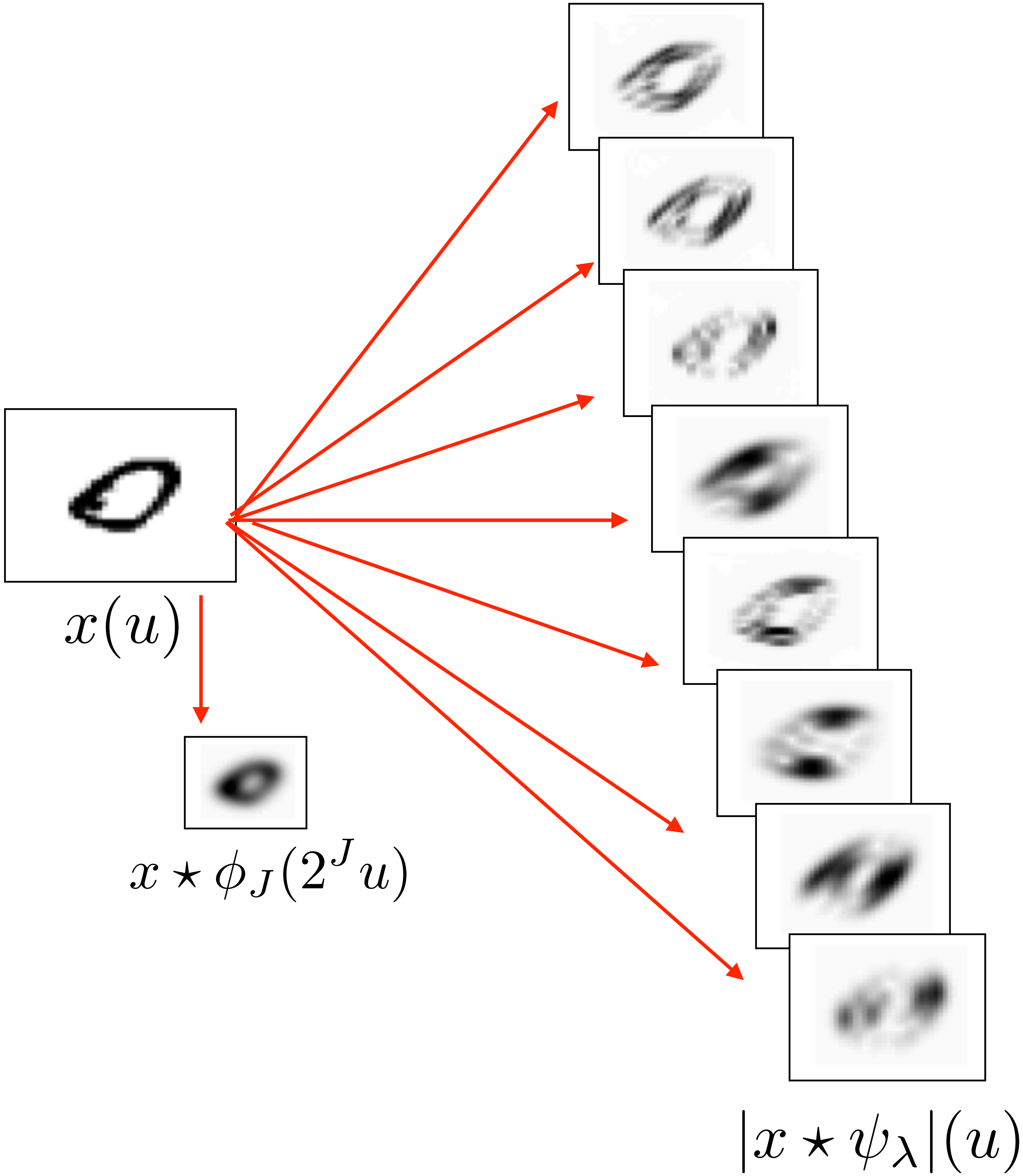
Cascade of contractive operators.

Scattering Example



$x(u)$

Scattering Example



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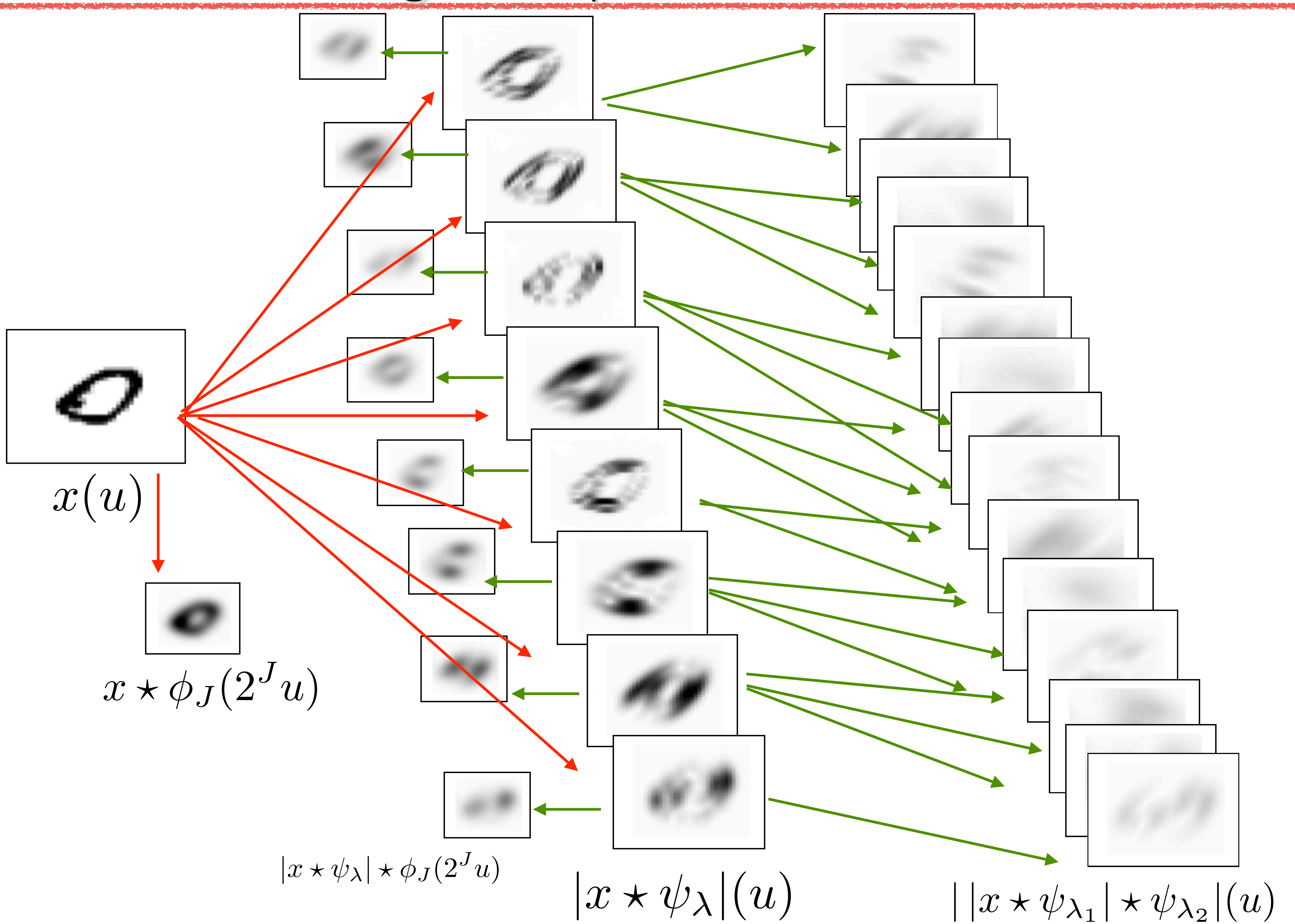

Scattering Properties

- Additive stability and conservation of energy:

Theorem *(Mallat)*: For appropriate wavelets, the scattering representation is contractive, $\|S_J x - S_J x'\| \leq \|x - x'\|$,
and unitary, $\|S_J x\| = \|x\|$.

$$\|S_J x\|^2 = \sum_{p \in \mathcal{P}_J} \|S_J[p]x\|^2$$

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- In practice, the transform is limited to a finite number of layers m_{max}

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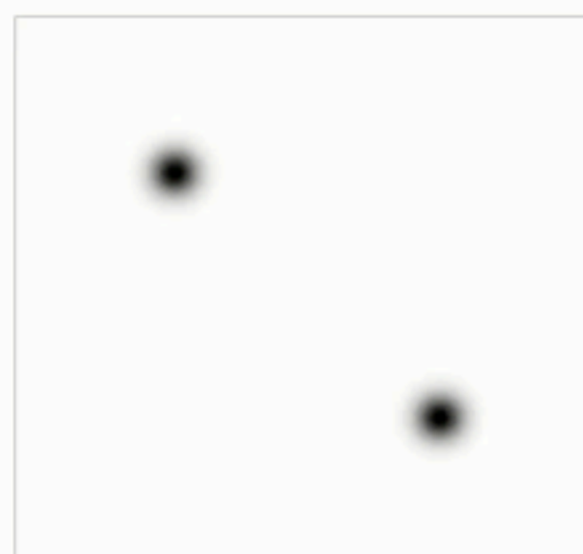
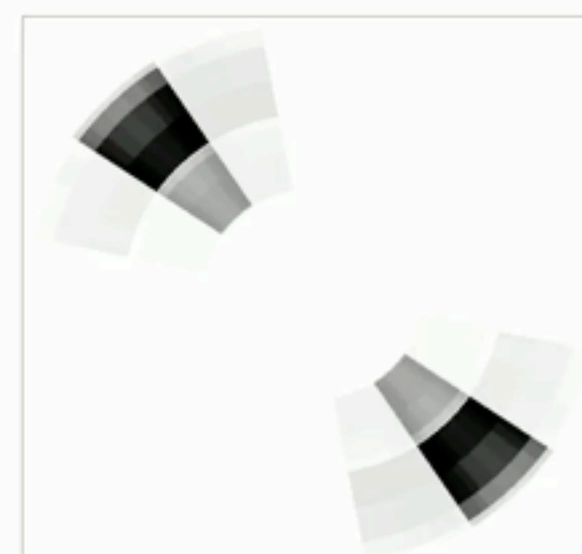
- Geometric Stability:

Theorem *(Mallat)*: There exists C such that $\forall x \in L^2$ and all m, the m-th order scattering satisfies $\|S_J L[\tau]x - S_J x\| \leq C m \|x\| \left(2^{-J}\|\tau\|_\infty + \|\nabla\tau\|_\infty + \|H\tau\|_\infty\right)$.

$L[\tau]x$



$|\widehat{L[\tau]x}|$



$S_J L[\tau]x$

Discriminability

- For appropriate wavelets, the information is preserved at each layer:

Theorem: (Waldspurger) For appropriate wavelets, the operator $Ux = \{x \star \phi_J, |x \star \psi_j|\}_{j \leq J}$ is injective.

- However, the inverse is unstable —> we might be contracting too much in general. How to prevent that?

- Sparsity In terms of contraction it is very intuitive.

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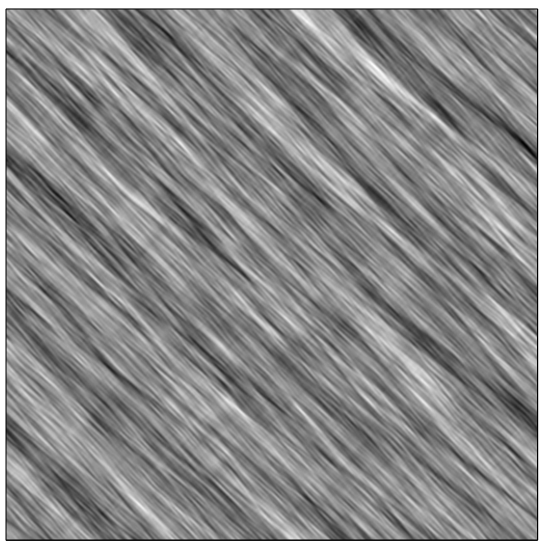
- However, if x, x' are sparse, this inequality is an equality in most of the signal domain.

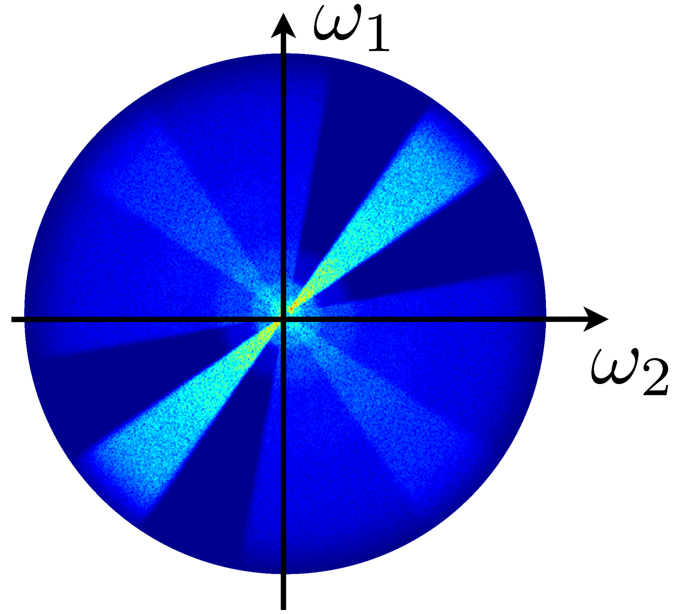
- Thus sparsity is a means to control and prevent excessive contraction of different signal classes.

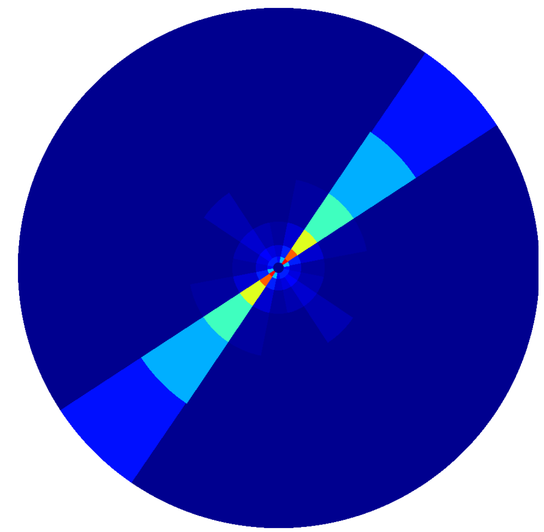
Image Examples

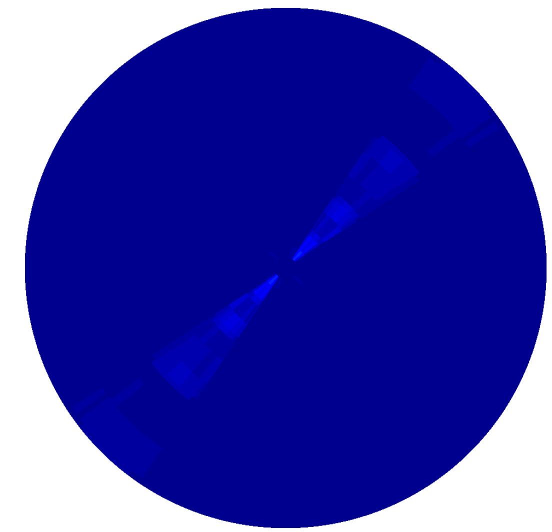
[Bruna, Mallat, '11,'12]

Images	Fourier	Wavelet Scattering	
f	$\hat{f}$	$\lvert f \star \psi_{\lambda_1} \rvert \star \phi$	$\lvert\lvert f \star \psi_{\lambda_1} \rvert \star \psi_{\lambda_2} \rvert \star \phi$

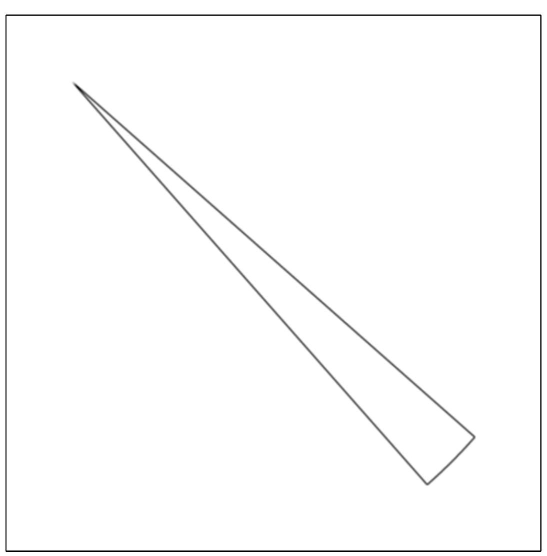


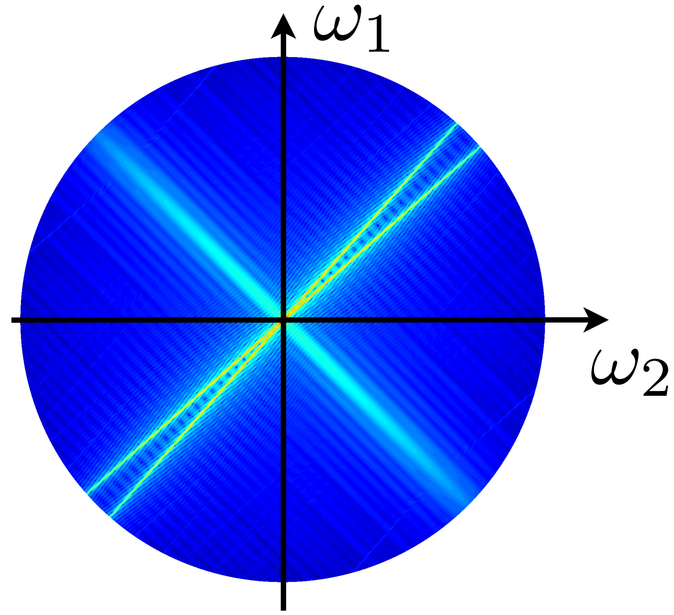


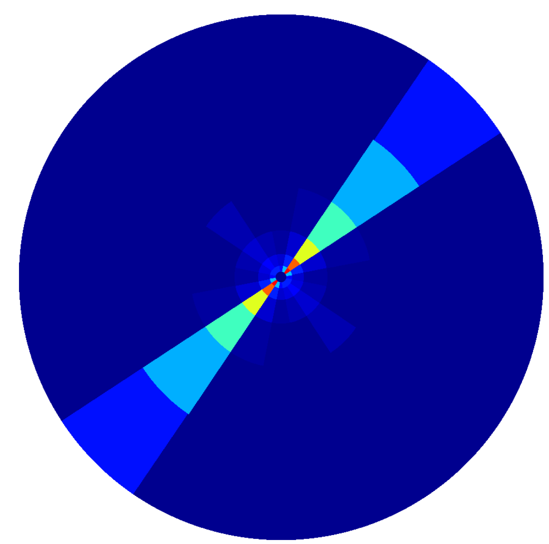


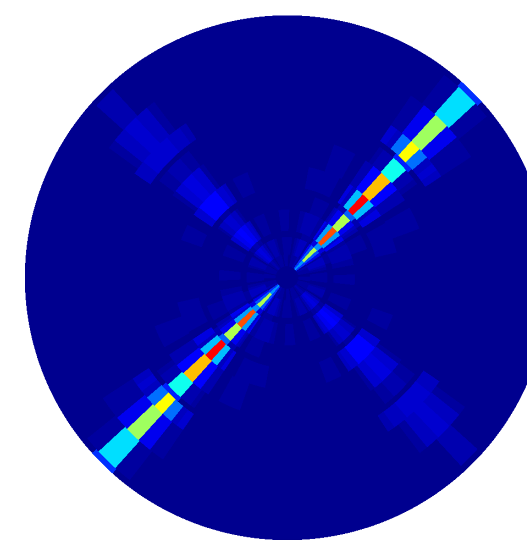


SIFT





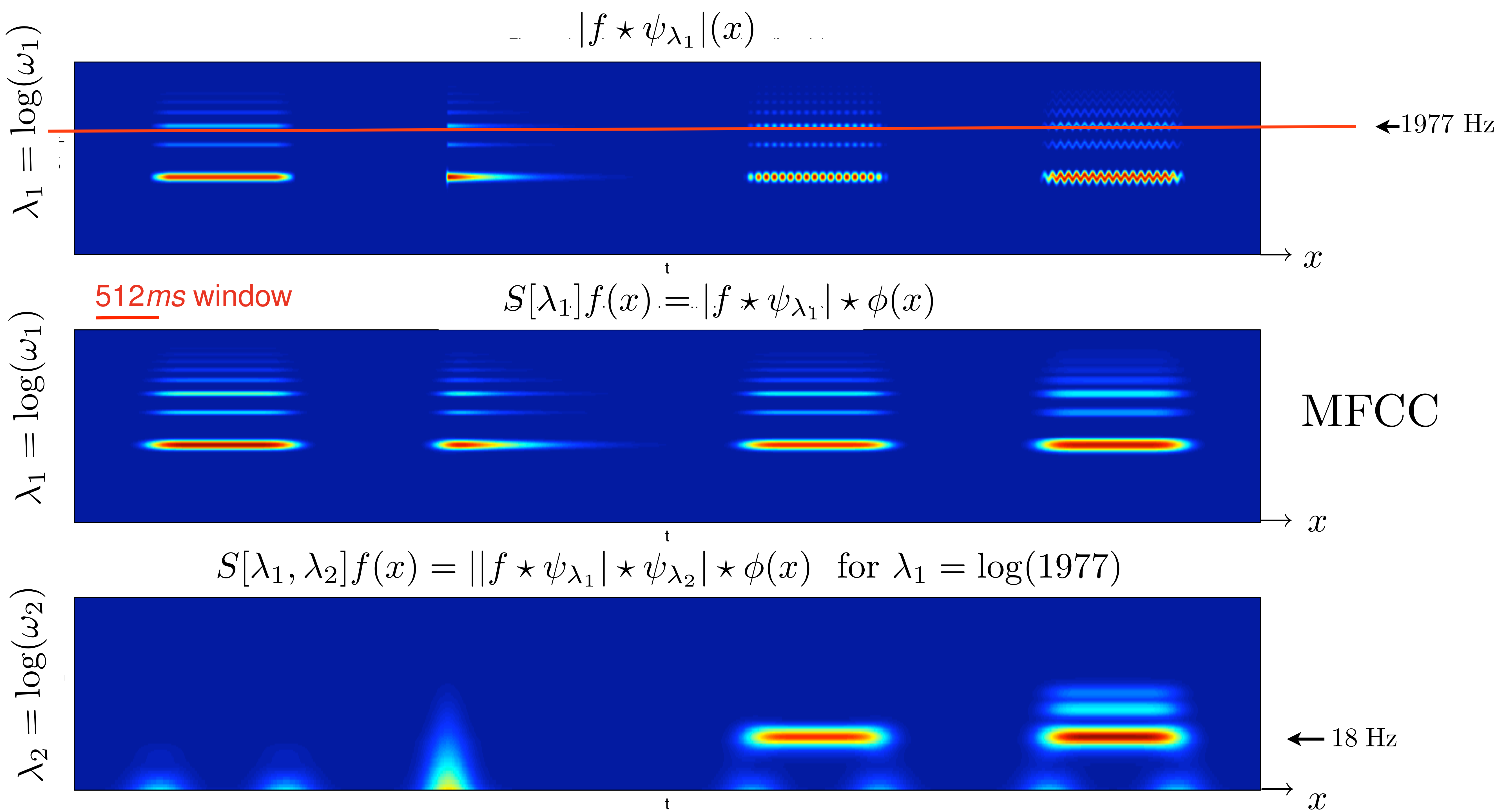




window size = image size

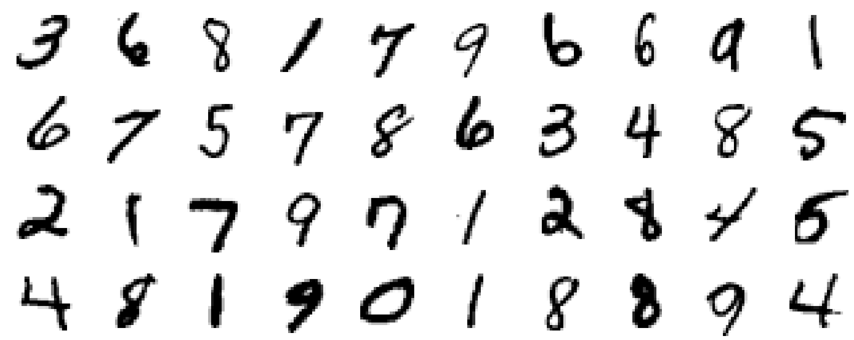
Sound Examples

(courtesy J. Anden)

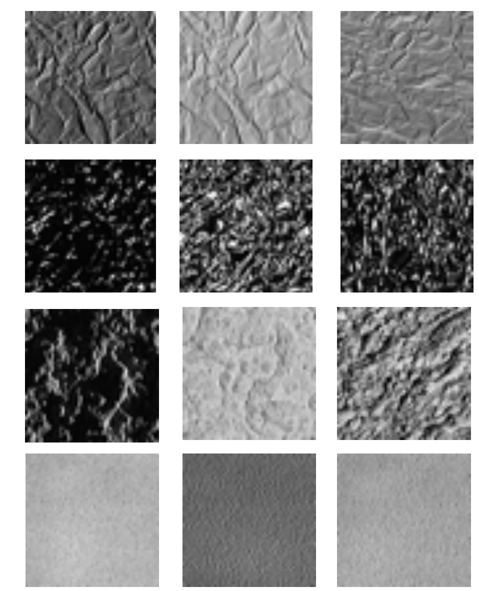


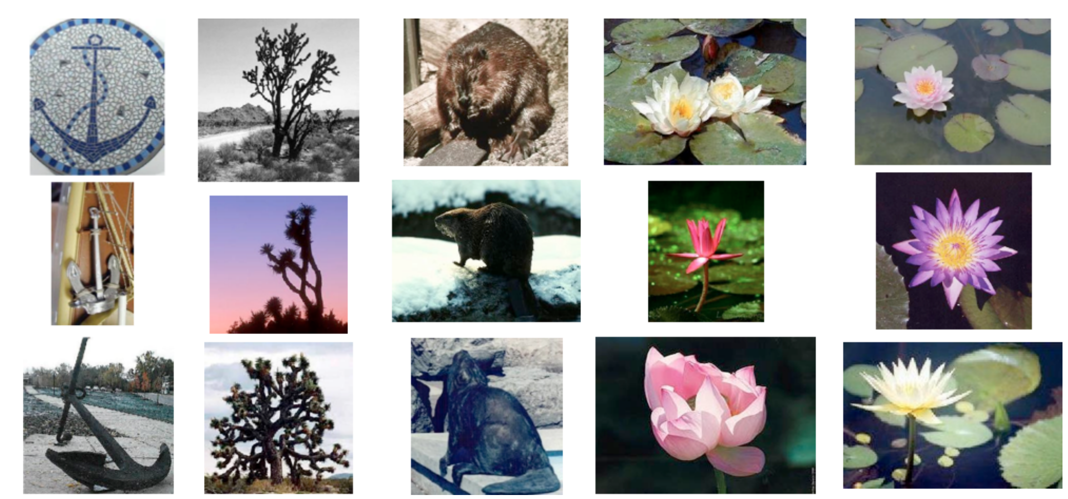
Classification with Scattering

- State-of-the art on pattern and texture recognition:
  - MNIST, USPS [Pami'13]



  - Texture (CUREt, UIUC) [Pami'13]





- Object Recognition:
  - ~17% error on Cifar-10 [Oyallon&Mallat, CVPR'15]

Limitations of Separable Scattering

- No feature dimensionality reduction
  - The number of features increases exponentially with depth
- Feature maps are not recombined
  - The deformation model is inherited from the input domain: we will see that recombining feature maps offers more powerful invariance.
- Feature maps are not learnt
  - We shall see that adapting the filters to object classes improves contraction AND discriminability.